THE MATHEMATICS TEACHER

Volume XLII MAY 1949 Number 5

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OFFICIAL JOURNAL OF THE
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

THE MATHEMATICS TEACHER

Official Journal of the National Council of Teachers of Mathematics

Devoted to the interests of mathematics in Elementary and Secondary Schools

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THE MATHEMATICS TEACHER

Volume XLII Number 5

Edited by William David Reeve

Polar Maps

By JOHN KINSELLA
Ohio State University, Columbus, Ohio
AND
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IT IS time for us to get better acquainted with the top and bottom of the earth. The strategic importance of the North Polar Regions is becoming increasingly evident and recent explorations in Antarctica have focused attention on that part of the world. We are accustomed to looking at maps which either exclude the polar regions or which distort excessively the distances, directions and relative size of areas in these parts of the globe. Many maps in common usage do not indicate clearly that the great circles between many important cities in the Northern Hemisphere pass near the North Pole.

The aim of this paper is to suggest ways in which a study of polar maps can contribute to certain objectives of mathematical education. The fact that polar maps show the importance of mathematics to air age geography should be a means of stimulating student interest. Although interest alone is not sufficient to make school learning valuable, the ability to apply mathematics to problems in social and scientific fields is an important educational aim. Frequently such problems require combinations of various elements from different branches of mathematics. The ability to select the mathematical equipment necessary for a solution can only be developed by experience. A study of polar maps provides just such experience, for the students are compelled to use arithmetical computation, algebraic processes, geometric principles and trigonometric functions.

In the development of polar maps teachers have an opportunity to reveal the structure of mathematical thinking. Keyser has repeatedly emphasized how a slight change in a group of independent assumptions frequently leads to a considerable change in the derived conclusions. For mathematicians this statement is a truism; the products of our thought are bound to be a function of the conditions originally accepted. A map which has the property that any area on the sphere and the corresponding area on the map have a constant ratio yields a vastly different picture from one in which great circles are represented by straight lines. Which represents the "true map?" One answer is "neither" for no flat map will be a correct picture of a spherical surface in all respects; another answer is "both" since each map has a certain definite property and has a proper application for a certain definite problem.

Nature of Mapping

The general problem of global mapping is admirably indicated in the following quotation: "The beginning of wisdom in cartography is the realization that all maps *without exception* are distorted. It is a melancholy reflection on the state of our geographical sophistication that it is still necessary to emphasize this simple fact. All maps have distortion and on most maps worth the name the distortions are present in order that the map may be true in one particular respect."* The mathematical description of these distortions requires, in many instances, far more mathematical background than can be provided by the usual high school curriculum. Therefore, it is important to exercise care in selecting maps for study. Fortunately, polar maps may be satisfactorily developed by the methods of secondary school mathematics.

The problem of map projection is to determine a systematic method of representing on a flat surface the meridians and parallels of the earth. This is equivalent to establishing a one-to-one correspondence between the points of a plane, the map, and a sphere, the earth. Thus, if latitude is denoted by L and longitude by λ, the point (L, λ) on the sphere is mapped by the point (x, y) where

$$x=f(L, \lambda) \quad \text{and} \quad y=F(L, \lambda).$$

In some cases the map is constructed by graphic means without explicit calculation of the x's and y's. It should be noted, too, that the term "projection" as used in mapping does not always imply a projection in the geometric sense.

Four polar maps are described in the following discussion. Formulas are derived where appropriate, and each map is illustrated by a figure in which certain well known cities, and the great circle routes connecting them, are shown. Only on the gnomonic map do these great circles appear as straight lines.

Polar Gnomonic Projection

Description: This is a geometric projection from the center of the sphere on a plane tangent at the North Pole. The meridians are mapped as equally spaced straight lines radiating from the Pole, since the intersections of the planes of the meridians and the tangent plane are straight lines passing through the Pole. The parallels of latitude are mapped as circles on the tangent plane. The problem of constructing this map is simply that of determining the radii of these circles. In Figure 1, O is

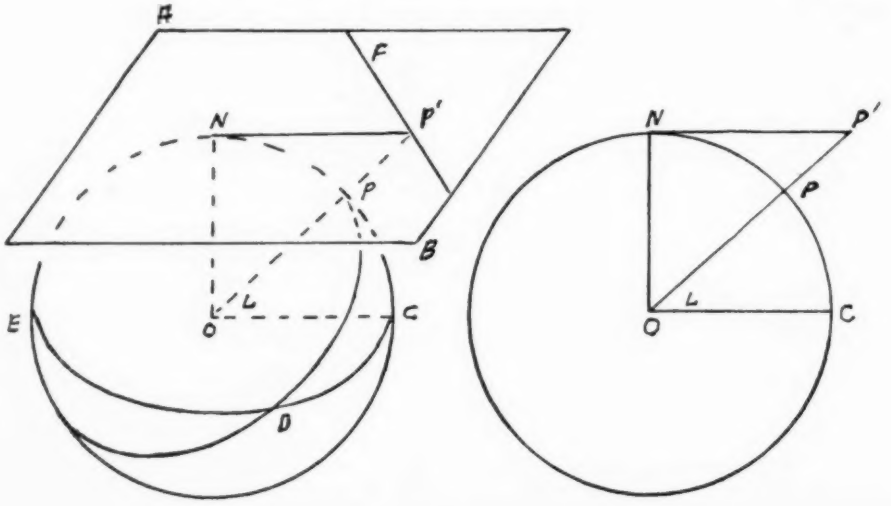

Fig. 1

the center of the sphere, N is the North Pole, AB is the tangent plane at N, P is any point of the Northern Hemisphere, L is the latitude of P, CDE is the equator, and NP' is the intersection of the plane of the meridian through P with the tangent plane. Let $NP' = R$ and r the radius of the sphere, then $R = r \cot L$. All points on the parallel of latitude through P will have equal R's, and the parallel is mapped by a circle of radius $r \cot L$. In other words, the parallels are mapped by concentric circles with radii proportional to the cotangents of their latitudes. Figure 2 represents a polar gnomonic projection from latitude 40 to 90.

Extent: From the nature of the projection it is obvious that the area which can be mapped will be less than a hemisphere.

Property: Since the intersection of two planes is a straight line, the map of any great circle, say PD, will be a straight line, FP'. Conversely every straight line on the map represents a great circle on the sphere.

Use: Since areas, distances, directions, and shapes are violently distorted the gnomonic projection has no use except the very important one of showing great circle routes. In navigation the latitude and longitude of points on the great circle track have to be transferred to a different type of map.

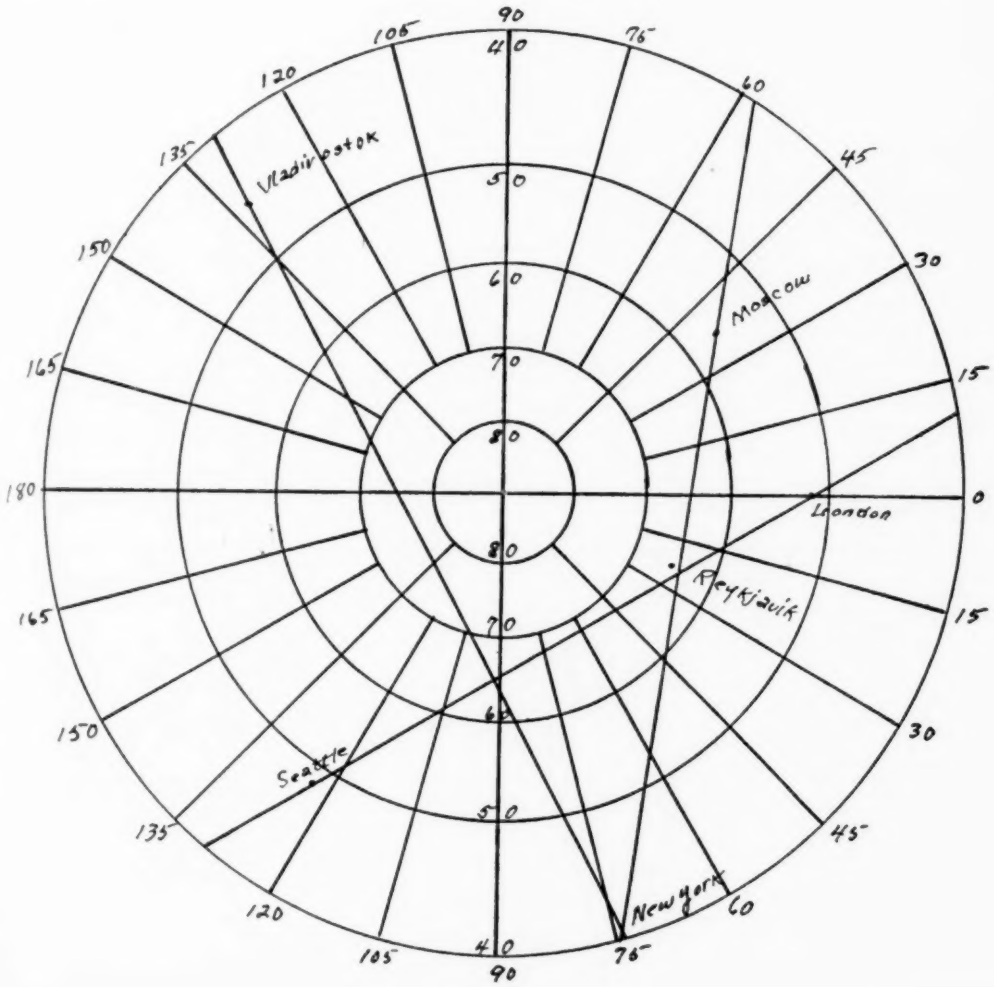

FIG. 2

POLAR STEREOGRAPHIC PROJECTION

Description: This is also a geometric projection with the center of projection at the South Pole (Figure 3). Since arc NP equals $90-L$, angle NSP' equals $90-L/2$. Hence, $NP'=2r\tan(90-L)/2$. Figure 4 is a polar stereographic projection of the Northern Hemisphere. As in the gnomonic projection the meridians are mapped as equally spaced straight lines radiating from the North Pole; and the parallels of latitude are circles with center at the North Pole and radii proportional to $\tan 90-L/2$.

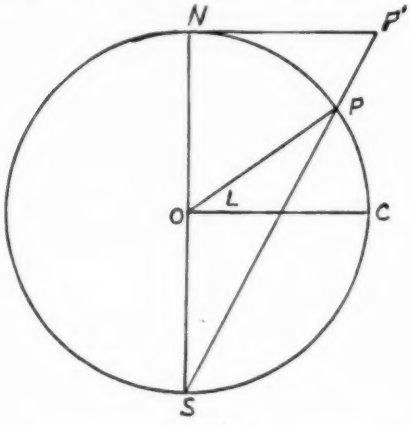

FIG. 3

Extent: The whole sphere may be mapped, but since the latitude circles in the Southern Hemisphere are magnified tremendously, areas in this hemisphere are violently distorted. Hence it is not desirable to extend the map beyond the equator.

Properties: The stereographic projection

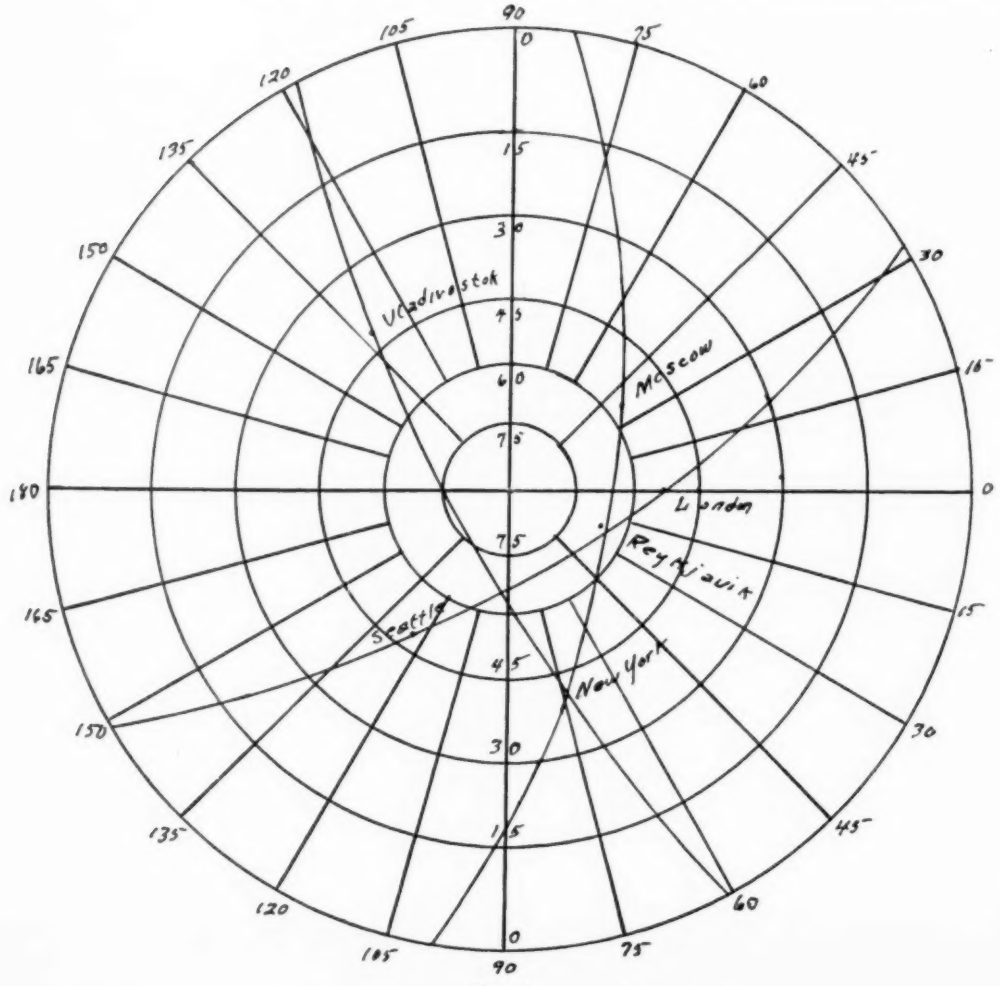

FIG. 4

is conformal. This means that if C_1 and C_2 are two curves on the sphere, which are mapped by two curves G_1 and G_2 on the map, then the angle between the former curves equals the angle between the latter. Conformality may also be described by saying that at any point of the map the scale is the same in all directions, even though the scale may vary from point to point. The three other maps presented here are not conformal. Another distinctive property of this map is that every circle on the sphere, which does not pass through the South Pole, is mapped as a circle; circles passing through the South Pole are mapped as straight lines.

Uses: The stereographic projection is a good general purpose map for a hemisphere. In the vicinity of the North Pole straight lines on the map are close approximations to great circles, so that this projection has uses in polar navigation.

Polar Azimuthal Equidistant Projection

Description: This is not a geometric projection. The meridians are again equally spaced straight lines radiating from the North Pole. The parallels of latitude are mapped as *equally spaced* concentric circles with the Nroth Pole as the center. If R_E is the radius of the map of the equator and R_L the radius of the map of any parallel of latitude, then $R_L = R_E(90-L)90$. Figure 5 shows this projection.

Properties: Distances along the meridi-

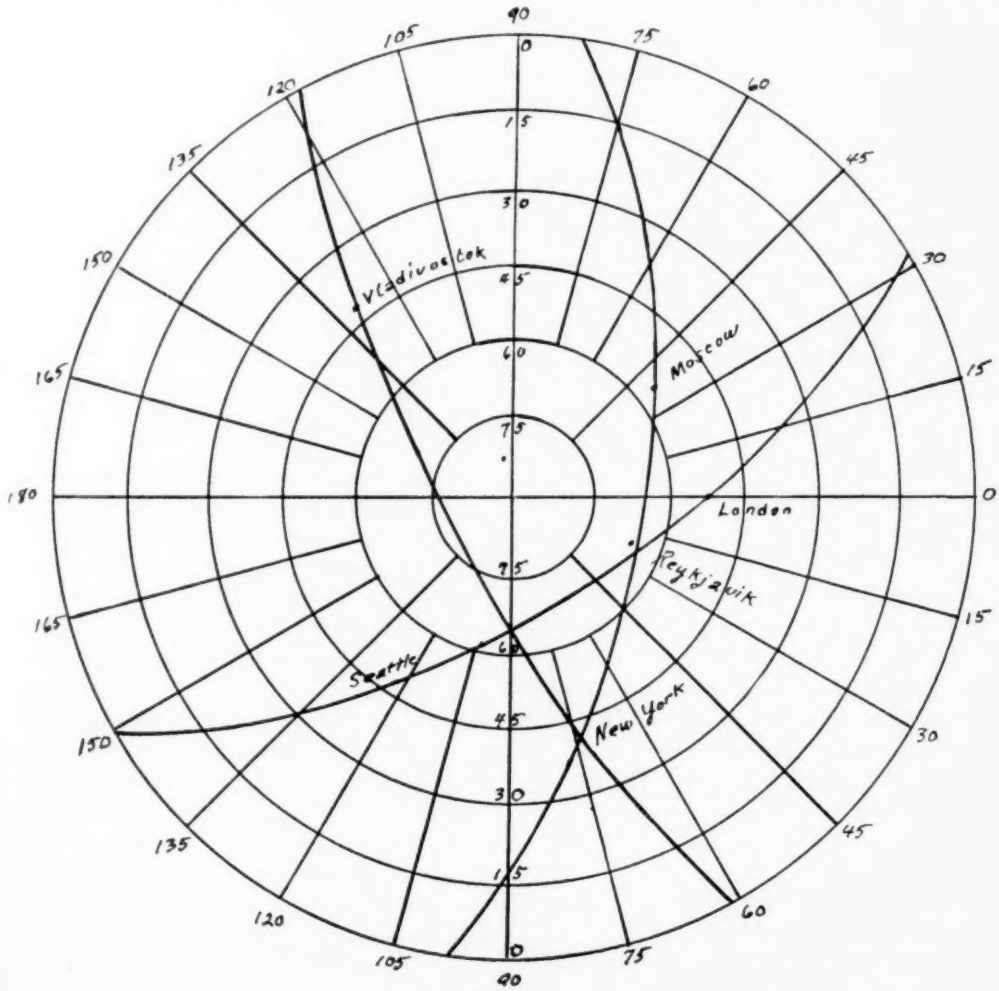

Fig. 5

ans are in the same ratio as the corresponding distances on the sphere and directions from a single point, the North Pole, are correctly mapped. The map has no other properties.

Extent: The whole sphere may be included although there is great distortion of size and shape especially in the Southern Hemisphere. The South Pole is mapped as a circle with radius twice the radius of the equatorial circle.

Use: This is also a general purpose map. When some point other than the Pole is used as a center, great circle distances and true great circle bearings from this center are correctly mapped.

Polar Equal Area Projection

Description: The polar equal area map is not a geometric projection. The meridians are of course equally spaced straight lines

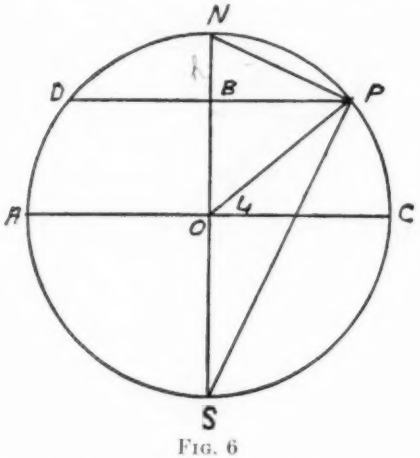

Fig. 6

radiating from the Pole, and the parallels are concentric circles. The problem is to determine the radii of the concentric circles so that any polar zone will have the correct area. Figure 6 represents the plane of a meridian; N and S are the poles, O is the center of the earth; PD the intersection of the meridian with the plane of a parallel of latitude, and AC the equatorial diameter.

Angle $POC = L$, the latitude of the parallel through P and D.

Let r = radius of the sphere, and R = radius of the circle mapping the parallel.

Area of polar zone $= 2\pi rh$.

In triangle OBP: $OB = r \sin L$

$h = r - r \sin L = r(1 - \sin L)$

Area of zone $= 2\pi r^2 (1 - \sin L)$

Area of circle on map $= \pi R^2$

$$R^2 = 4r^2 \frac{1 - \sin L}{2}.$$

Applying the formula for the sine of half an angle,

$$R = 2r \sin \frac{90 - L}{2}.$$

The above formula for R is the equivalent of a geometric theorem: the radius of the circle mapping a parallel of latitude equals the chord of the complement of the latitude. That is, in Figure 6, $NP = R$.

In triangle NSP: $\overline{NP}^2 = \overline{NB} \cdot \overline{NS}$

$\overline{NP}^2 = 2rh$

$\overline{NP} = R$

Extent: The whole earth can be mapped on this projection with increasing distortions of shapes and directions in the Southern Hemisphere.

Uses: Equal area projections find their proper use in the presentation of such statistical information as distribution of population, natural resources, etc.

Classroom Use

The study of polar map projections requires an understanding of some elements of spherical geometry and plane trigonometry. In most high schools of the United States this dependence would probably defer the teaching of this material to the twelfth grade. However, with the use of appropriate visual aids it would seem possible to present the gnomonic and stereographic projections in the junior high school, without of course giving the mathematical development. It is obvious that the reading and interpretation of maps does not require the same amount of mathematical maturity as does the derivation of the formulas underlying their construction.

The various kinds of distortions mentioned above cannot be discussed rigorously without the aid of more advanced mathematics. However it is possible to discuss some of these distortions in an informal manner. Such discussion should probably not be attempted before the eleventh or twelfth grades. For example, the area distortion in the gnomonic and stereographic projections can be shown by the use of the function idea. It is apparent that the areas of polar zones on the sphere vary directly as $1-\sin L$ while the areas of the circles mapping the parallels on these two projections vary as $\cot^2 L \tan^2 90-L/2$. The distortion of distances can easily be illustrated in the case of the gnomonic projection. On the sphere the distance of a point from the North Pole is proportional to $90-L$, since the arcs vary as the central angles. On the map however the distance of a point from the pole varies as $\tan (90-L)$. Since the angle is not proportional to its tangent, distortion of distances is "proved." This functional approach and comparison of changes on the map plane with changes on the surface of a sphere can be used with the other projections, to show approximately and intuitively that the distortions do occur.

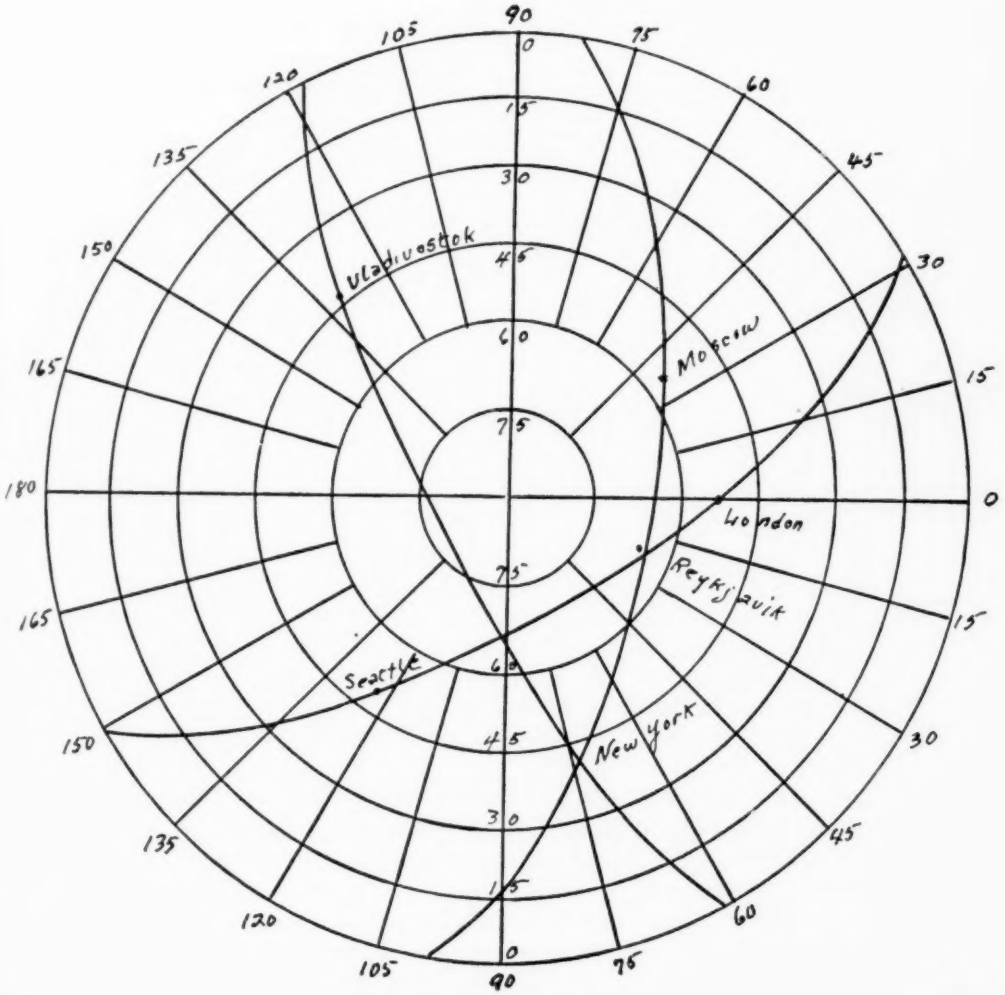

FIG. 7

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A Mathematics Course for any Student

By John Schmid, Jr.
University of Wisconsin

The Commission on Post-War Plans in its first report proposed three series of mathematics courses. The sequential courses would consist of algebra, geometry, and trigonometry. Related courses, growing out of improved general mathematics courses, would attempt to meet the needs of students entering industrial trades. Social mathematics courses, providing a degree of functional competency in everyday affairs, would conduce toward greater mathematical literacy in American society. It would be impossible to put into effect a program as extensive as this in most secondary schools because many classes would be necessary. A plan, however, whereby small or medium sized secondary schools might carry out the Commission's proposal was tried at the University of Wisconsin High School during the spring semester of 1948. Dr. John R. Mayor, director of the mathematics program and course supervisor, believed that a 12th grade class, divided into groups, might be conducted in which these groups independently could pursue any of the proposed courses or diverse sequences of selected mathematical topics. In this way the course might be built around diverging needs, purposes, and interests of students. It was hoped that this techinque would be practicable for small secondary schools where prohibitive factors preclude many classes.

The course was appended to the regular mathematics program. One-fourth credit was allowed because class was held only three hours per week. Thirteen students enrolled for the course, eight girls and five boys. All had completed one year of algebra, eleven had completed one year of geometry, and two had completed second year algebra and third semester geometry. The latter two students currently were taking trigonometry.

The needs, purposes, and interests of the students were identified in several ways. On the first day of class, the students and teacher discussed values that might be obtained from the course. The teacher made a brief survey of topics from arithmetic, algebra, geometry, and trigonometry. This survey was intended to suggest to the class concepts that might tie in with the students' needs, purposes, and interests. It was learned that all members of the class expected to go to college. The students expressed the desire to review and survey those mathematical topics that either would prepare them for entrance examinations or subsequent college work.

The Thurstone Algebra Test was administered on the second day of class. Despite the fact that all students had completed one year of algebra, the majority had difficulty with elementary processes and concepts. This made possible a tentative guide for the course.

Another way of identifying student needs was by means of a checklist of the mathematical competencies formulated by the Commission on Post-War Plans. Each student was asked to check those competencies that he felt he possessed as well as those in which he felt inadequately prepared.

A checklist of purposes for taking the course was prepared by the teacher with the aid of the course supervisor. Each student checked those that pertained to his reasons for taking the course. It was felt that the identification of needs and purposes by many methods would aid in the development of a sound mathematics program. As new needs and purposes emerged during the semester, the nature and implications of these needs and purposes were discussed in the classroom. In this way subject matter content at all times was related closely to the students' goals.

Although all the students expected to go to college, different degrees of mathematical preparation and varying interests furnished a basis for separating the class into groups. A group of three boys wanted to study topics from advanced algebra and trigonometry. Another group of eight students wanted to review elementary algebra and geometry. In addition to this review, the group studied concepts which were new to them. During the semester, they spent time on exponents, logarithms, the slide rule, graphing, reading various types of mathematical tables, geometrical constructions and proofs, equations, simple trigonometry, and related topics. A third group of students, consisting of two girls, initially preferred to confine their work to arithmetical concepts and processes. Towards the middle of the semester, these girls occasionally joined the group of eight students when topics arose from which the girls could profit.

The group lines were not rigidly fixed. At times students passed from one group to another; and occasionally a student worked on a self-chosen topic by himself. After completing the assignment, the individual returned to his group. Catching up to the group seemed to cause no great difficulty. This might have resulted from having the students within each group work cooperatively.

All three groups temporarily were united from time to time when a topic common to all was presented. For example, a diagnostic test in arithmetic indicated that all members of the class either were slow in computing or were deficient in some arithmetical processes and concepts. Every few days ten to fifteen minutes were devoted to a review of essentials and simple computations. The students were quite pleased to learn shortcuts that accelerated their skills in fundamental operations.

It was found that the teaching process involving three groups was best conducted by spending ten to twenty minutes each period with each group separately. Any remaining time during the period was given to individual help.

Within each group the members worked cooperatively. Although the students turned in their individual work, they were encouraged to help one another over difficulties. This seemed to work satisfactorily, for if a student had difficulty with some problem, he could work with someone having the same trouble, instead of idly waiting for help from the teacher who might be busy with another group. At all times, emphasis was placed on learning something new, rather than just getting the work finished correctly.

Student needs, purposes, and interests served as a rough guide for the selection, scope, and sequence of subject matter content. A text was selected, but it was realized at the outset that it would serve as a reference book only. Numerous other books from the school library were used extensively throughout the course. Supplementary mimeographed material in the form of exercises and problems was handed out frequently. The preparation of this material required a great deal of time, but this was found to be the most expedient way of providing exercises suited to student needs.

An evaluation of the experiment indicated that the course did contain sufficient merit to warrant its being repeated the following spring semester. The supervising teacher and the teacher were encouraged by the outcomes and satisfaction expressed by the students throughout the semester. However, it was felt that better results would have been secured had the class met five times per week rather than only three times. The teacher felt that the workability of this type of course is within practical limits for the average teacher. Other mathematics teachers had predicted jokingly that the class would be a three-ring circus. At times it did seem like this, but not frequently. The course required far more of the teacher's time for preparation than a conventional class would have required. It is believed, however, that as a teacher gains experience in conducting this type of

course, the time and preparation elements will be minimized. Administrators interested in innovating similar courses in their schools would be wise in relieving the teacher from work in some other area, at least for the first trial. Unless the teacher has ample time in the initial trial, the possibility of success seems unlikely. There is ever present a tendency for the sequence of subject matter content to progress haphazardly. However, a basis for continuity of the learning process lies in a continuing reference to the students' needs, purposes, and interests. As the teacher acquires experience in guiding diverse groups simultaneously, he can expect improvement in the course.

A testing program is recommended at the beginning of the semester in order to identify student needs and to serve as a means for separating the class into groups. Testing should cover all fields of elementary mathematics so that a total picture of the student's mathematical needs is obtained.

Administrators of small secondary schools might find a course of this type quite practical. It can extend the school's offerings so that the needs of all students may be met. The course should not be an administrative expediency for lumping together students of low ability or students that are problems in other courses. Its purpose has origin in meeting students' needs; therefore, it is a course for any student.

The rapid rate at which mathematics is growing makes it almost impossible for teachers to keep from becoming antiquated. Most of us have neither time nor energy to study courses beyond the fields in which we are teaching.

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MATHEMATICS MAGAZINE

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Dimensions for High School Students

By Israel Wallach
Thomas Jefferson High School, Brooklyn, New York

The word *dimension* has won considerable popular appeal in recent years. The purpose of this article is to show some of the uses of the idea of dimensionality in the classroom as an aid to teaching and to a greater understanding of mathematics by the students.

To define dimension is difficult. Nevertheless, students like to discuss dimensionality and to deal with the idea in a very concrete way. The need for an exact definition does not trouble them very much. I should like to present here an approach that I have used and which the students seemed to like. At any rate, it is desirable that at some point in high school mathematics there be a lesson or two on the subject of dimensions.

A most appropriate place for discussing the idea of dimensionality would be the beginning of the solid geometry course. It could be useful in the course in plane geometry. Teachers may find it useful at other points in high school mathematics. I have tried it with ninth-year classes in Ingrated Mathematics as introductory and orientation lessons in high school mathematics. Students can learn right at the beginning the dual nature of mathematics: It deals with phenomena in the world from their quantitative (numerical) and qualitative (spatial) aspects.

Once the idea of dimensionality has been taken up, frequent reference to it for clarification of new ideas under discussion can be made. For example, in geometry the *degree* of the formula for a length, area or volume can be a means of verifying the formula the student is using. An area formula has to be of the second degree since an area is two-dimensional. In $K=bh$ or s^2, bh or s^2 are expressions of the second degree. For the triangle $K=1/2\ bh$, $1/2\ bh$ is of the second degree and the $1/2$ is called a *constant,* and we introduce thereby another useful concept. A perimeter formula has to be of the first degree, e.g, $p=4s$, $p=2l+2w$ or $p=a+b+c$. Pupils who confuse πr^2 with $2\pi r$ will be helped. The first expression is of the second degree and therefore the area, K is equal to πr^2 while the circumference formula is $c=2\pi r$, with π and 2π as constants in the respective formulas. As another illustration: The degree of the ratio of corresponding lines, surfaces and volumes of similar figures will agree with the dimension of the spaces being compared. Thus, if the ratio of corresponding lines in two similar figures is $a:b$, the ratio of corresponding surfaces (two-dimensional aspects of the figures) is $(a:b)^2$, of volumes (three-dimensional aspects) is $(a:b)^3$.

In some classes, the students wanted a definition of the word dimension and tried to formulate their own. Considering a dimension equivalent to *a degree of freedom* in space generally seemed to satisfy them. Almost all students readily accept the right angle as the angle between coordinate lines in two or three dimensions, although mention should be made that this practice is purely a convention. There is, also, the familiar difficulty in the students' attempt to *visualize* a fourth coordinate line for four dimensions at right angles to the other three lines. For students who would like to delve more deeply, we would have to offer suitable references, some of them above the secondary school level. In Murnaghan's *Analytic Geometry* published recently we find these statements about dimensions: "The maximum number in a set of linearly independent vectors on a line is *one;* while the maximum in a set of linearly independent vectors in a plane is *two.* That is what we have in mind when we say that a line is *one-dimensional* or *possesses only one dimension* while a plane is *two-dimensional* or *possesses two dimensions.*"[1] Later

[1] F. D. Murnaghan. *Analytic Geometry.* New York, Prentice-Hall, Inc., 1946. Page 60.

he states, "Any space vector may be expressed as a linear combination of any three linearly independent space vectors."[2] His remarks about the geometric ideas in the solution of algebraic equations are worth considering. "You must thoroughly dispel from your mind any idea that one must be some kind of mental wizard to deal with geometric ideas in space of four, five, six or more dimensions. If you know how to work with vectors in a plane or in space (of three dimensions), the methods available in these cases carry over, with very little modification, to spaces of higher dimensions (that is, to the cases where the vectors involved have more than three coordinates)."[3]

In the classroom discussion, we concentrate on the dimensions from zero to three, with some allusions to other dimensions. The following table is developed during the discussion:

Dimension or Number of the space	Usual Name	Name of Its Size
0-space	Point	(No size)
1-space	Curve	Length
2-space	Surface	Area
3-space	Solid	Volume

The discussion can be initiated in either of two ways. If we have outlined geometry as a science and have mentioned the need for undefined elements, we can consider space as an undefined element and then name the spaces as given above. A more concrete way of beginning would be to ask the students in how many dimensions they believe they live or move. They generally say three. (In a very good group we could ask, "Why?" and begin a discussion of a definition of dimensionality.) Since the two-dimensional coordinate system is familiar to them, they readily accept the need for three coordinate axes as a means of orientating themselves in three dimensions. The first three fingers of one's hand can be held up in the classroom at approximately right angles to each other to illustrate the three degrees of freedom within the room or as a means of locating any point within the room with reference to the coordinate system thus set up. Other means can be used, such as a corner of the room, three sticks in a Tinker-Toy wheel at right angles to each other, etc. By contrast, we can use the blackboard or the top of the desk to illustrate a surface or two-space with two coordinate lines. We ask if a ball or any solid is a three-space or two-space figure. The students will make the distinction between the interior of a solid and its surface. Here, reference can be made to the coordinate system on the earth's surface. Similar discussions can follow for the one-space, curve or line, and the students will distinguish between the surface of a circle and its circumference. (Some students tend to consider a quadrilateral a four-space but other students will correct this.) Illustrations of one-dimensional coordinate systems can be requested and we receive in response, the ruler, protractor, thermometer, clock, etc. From the discussion outlined above the students learn that the number of dimensions of a given space is equal to the number of coordinates needed to locate a point in that space.

We can now proceed to develop the table given previously. List the zero-space as a point. The fact that the point has no size but does have position is helpful to the students, especially in the beginning of the study of geometry. Move a point, using chalk on the blackboard to illustrate, and not as in a spiral which will be confusing, and a one-space is generated. By moving the curve and joining the corresponding points of the initial and final positions of the curve, a two-space or surface is generated. (Again, any spiral motion or self-intersecting motion should be avoided.) The generating curve does not have to move in a plane. By moving a two-space a three-space is generated. We may now say at this point, "And so on," for the higher spaces, if the class will understand what is

[2] Same. Page 125.
[3] Same. Page 161.

meant. A model or picture of a tesseract would come in handy. We might refer the students to Chapter VII on "Third and Fourth Dimensions," in Swenson's *Integrated Mathematics*, Book II. The motions for generating the spaces show their mutual interdependence. To emphasize this interdependence we can reverse the procedure. The end (boundary) or the cross-section of a three-space is a two-space, of a two-space a one-space, of a one-space a zero-space. Any space contains in it all the lower spaces. Thus, on or within any solid there can be located surfaces, curves or points.

As a test of the students' grasp of the notions discussed, we can request them to mention objects which move in one-space, two-space of three-space. They can compare the motion of a train, an auto, the airplane, an elevator, boats, submarines and so on, and the dimension or degree of freedom that each has. An interesting point to raise would be to ask the number of facts or coordinates needed for making an appointment (1) at a well-known point, (2) at an intersection of two streets, (3) on a certain floor of a building at an intersection.[4] Another point that would be of interest to a senior group would be to ask the dimension of the time in a velocity or in an acceleration.[5]

What has been presented above is primarily for the purpose of giving students of mathematics a frame of reference that will assist them in their study. There are other purposes that can be served by the discussion of dimensionality during a student's high school career. The student is learning something that is consistent with his study of geometry and possible later work in the higher branches of the subject. He is being introduced to the study of the general and essential properties of space and the property of dimensionality is a basic property of space. Another purpose that would be met, to a degree, by an early discussion of dimensionality is one that has been urged by some mathematics teachers for a long time. It was alluded to earlier in this article when we spoke of the 'dual' outlook of mathematics. Teachers have urged that we teach the numerical and spatial relationships simultaneously. In line with this, I should like to offer a quotation from a lecture by Professor Oswald Veblen: "I do not advocate the use of any particular set of axioms in the schools but I do advocate the introduction of analytic methods in elementary geometry. . . The obvious thing to do is to recognize that we have a subject to present which has to do with physical reality. Let us approach it as any modern scientist would by first studying some of the observations and experiments with which we have to deal, in crude everyday terms. This would correspond, I suppose, to the "observational geometry" which is already in our schools. After getting a start in this way a scientist feels the need of a systematic language in terms of which to organize the phenomena. This language is mathematical analysis. In other words, as soon as the experimental basis is established, the study of geometry should proceed by the use of coordinates and the methods of analytic geometry."[6] This lecture by Professor Veblen contains many valuable suggestions for the teaching of mathematics. Teachers of mathematics will find it worth studying carefully. The discussion of dimensionality as proposed here, it seems to me, is a means of supplying some of the background that is required for a modern approach to geometry.

[4] See Infeld, Leopold, The Fourth Dimension and Relativity. A Radio Broadcast given by the Galois Institute of Mathematics, Long Island University, May, 1938.

[5] See Table I, page 27, War Department Manual T-M 1-400, *Theory of Flight*, February 27, 1941.

[6] The Rice Institute Pamphlet. Vol. XXI No. 4, October, 1934. Lecture entitled: "Certain Aspects of Modern Geometry." Page 215.

Fundamentals and Engineering Mathematics*

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1. INTRODUCTION

As was pointed out by the chairman, Professor Zant, I am a research scientist. You may therefore be skeptical about what a research scientist can have to say that would hold the interest of a group which is primarily interested in the teaching of high school mathematics. I shall try to put you at ease by informing you that I taught in a number of universities for eight years and that I do hold a Life Teacher's Certificate for the State of Michigan. Furthermore, I have a great personal interest in the problems connected with secondary education.

My association with engineers, research scientists, and other technically trained personnel added to my teaching experience has given me a deeper understanding of the problems of the education of these people. I hope that in the few minutes at my disposal I can convey my points of view on a few of these problems.

When I was first invited to address this organization I gave considerable thought to the choice of an appropriate title. The kernel of the theme had formed immediately but the choice of a few words to describe that theme was another matter. Those which were finally printed on the program do not entirely summarize the field that I should like to cover. Briefly, there are four points that I should like to discuss; namely,

1. Illustrations of basic mathematics in engineering and research.

2. Newly developed opportunities for a career in mathematics.

3. Guidance in steering youths into engineering and mathematics.

4. The teaching of fundamentals.

These are far-reaching points and, of course, I can't do justice to them. Undoubtedly you will not agree with some of the things I shall say and that is as it should be.

2. ILLUSTRATIONS OF BASIC MATHEMATICS IN ENGINEERING AND RESEARCH

The first point then is to present a few illustrations of basic mathematics in Engineering and Research. How often have you been asked, "But where do you ever use that?" This may not be as standard a question in high school mathematics as it is in college mathematics, but I know what you must have been asked that question. Many times when that question is asked, you are really hard pressed for an answer. For there are many concepts in mathematics that are not directly associated with common everyday things or events. Furthermore, if they are readily associated with practical things it is difficult to discuss that association in language that a high school student could follow. For example, you are teaching exponents in a high school algebra class. You are discussing the fact that $a \cdot a = a^2$, that $a^2 \cdot a = a^3$, etc. That mathematical concept is associated with the calculations necessary to determine whether or not an aircraft can fly faster than sound—but how are you going to explain that in his language? It is, of course, about impossible to do. I should therefore like to emphasize *that you should not spend a large amount of time explaining the connection between the concept and the illustration.* If that be necessary, the purpose of the illustration is lost and the illustration should not be used.

On the other hand, the ability to present vivid illustrations enables the instructor to create a motivation and hold an interest which otherwise is lacking. It is difficult

* An address delivered before the National Council of Teachers of Mathematics on December 30, 1948 at Columbus, Ohio. The opinions expressed herein are solely those of the author and do not necessarily reflect those of the U. S. Navy Department in any way.

for teen-agers to think along abstract lines. The fact that the "square root of a^2 is a" has little or no meaning; but if you can speak of it in terms of garden plots, swimming pools, badminton courts or airplanes, you are on familiar grounds and learning becomes easier. Garden plots and badminton courts, however, are "old-fashioned" and have been worked to death. The modern youth requires more imaginative and inspiring examples.

The most inpsiring example today is perhaps the airplane or associated flying missiles such as rockets. These are forever in the news and our great desire to attain higher speeds is fascinating. Fortunately the airplane offers many examples of elementary mathematics. A striking illustration which should create a motivation and maintain an interest would be to have an airplane either permanently drawn on the board or a picture of one and each time a new concept is encountered try to associate it with the airplane. Let us, for example, write the formula for the lift of an aircraft wing.

$$L = \tfrac{1}{2}\rho S C_L V^2.$$

This formula illustrates the use of symbols the concept of a formula, the use of exponents, and by replacing some of the letters by numbers it can be used to illustrate the solution of equations. Similarly, the drag formula and the speed formula can be used. I would even be in favor of displaying complicated differential equations such as the one showing the variation of the speed in a dive

$$\frac{dv}{dt} = \sin\gamma + \frac{1}{v} K_4 \cos\alpha - K_2 V^2 - K_3 V^2 \sin^2(\alpha+\alpha_0).$$

This formula can be used for illustrative purposes in algebra, geometry and trigonometry. You need not limit your attention to the airplane proper. It is also possible to use the equipment on the airplane such as the automatic pilot, which employs a gyroscope. This is a physical body. It has axes of symmetry; it consists of solids of revolutions, it has a large number of angles and it moves. It could be used in all kinds of illustrations in geometry.

In general it should be pointed out to all mathematics classes that motion can be expressed mathematically and from these expressions of motion come the designs of all of our modern machinery. That mathematics is the greatest tool known to mankind for the development and design of machinery.

There is one other point I should like to bring out. All mathematics classes should be narrated to. I think it is a good idea to spend a few minutes each day just talking about mathematics. And in those narrations be sure to point out that mathematics is the one subject which builds upon itself. That the start is made in arithmetic and that from there we go on to algebra, geometry, trigonometry, solid geometry, analytical geometry, the calculus, differential equations, and so on. I am in perfect agreement with the individual who objects to the repetition of the saying "you must learn this now for you will be using it later on." It may be a way of "passing the buck" for a reason why something should be learned. On the other hand it is exceedingly true in mathematics and in many respects it is also the beauty of mathematics, for nowhere else do you have such a beautiful sequence of development, and the student is entitled to be informed. "Calculus" and "differential equations" are not words which should be shunned. I think that high school students should be told that such things exist and it is better if he is shown where they fit into the scheme of mathematics.

3. Newly Developed Opportunities for a Career in Mathematics

The next point I should like to discuss, namely, "newly developed opportunities for a career in mathematics," is in a certain sense an illustration and it is the main theme that I should like to bring to you.

The boy or girl who wants to be an engineer—and there are many girls who are engineers now—can be told that it is important for him or her to be well versed in mathematics. Since mathematics is used in engineering, they will reconcile themselves to learning it because it is used in their profession and may some day help them to make a living. But the youth who is not interested in becoming an engineer may steer away from mathematics even though he likes it. He does not see where he could use mathematics as a profession except to be a teacher and, of course, he would hate to be a teacher. He couldn't own two cars if he were a teacher; maybe he couldn't even own one. So he steers away from mathematics, decides to become a lawyer, and the science of mathematics has lost a brilliant contributor. This youth, of course should be told of the new opportunities for mathematicians. However, I am really not so concerned about this individual; he may still become a mathematician. My primary point in discussing the opportunities for mathematicians is that I believe that the student's interest in the subject will be greatly strengthened and improved if he knows that mathematics is a profession; that it can be looked upon in the same sense as medicine, law, and engineering. If he knows that mathematicians are employed in work other than teaching he will "see more use for the stuff." He will respect the profession. It is because of this point of view that I hasten to mention that mathematicians are now being employed as mathematicians by industry, governmental agencies, research laboratories and scientific development institutes.

You may now raise the question "What do mathematicians do in industry and other places where they practice their profession?" And the answer is that modern design, development and research engineering has reached the place where it relies very heavily upon a so-called paper analysis before an idea is actually put into physical being. The old concept of taking an inventor's idea and immediately putting it into metal and then testing its practicability is still with us, but it must be admitted that it is not as prominent as it once was and certainly is being overshadowed by the paper analysis method, especially by the larger industrial organization. Now the paper analysis method takes on a form something like this:

(1) A thorough statement of the problems connected with the design or research;

(2) A transformation of the problems into the language of an analysis which can be done on a drafting board or at a desk;

(3) A large number of problems connected with a modern design dissolve themselves into the language of mathematics. These problems can then undergo a mathematical analysis, the results of which will appear in the language of mathematics;

(4) A transformation of the mathematical results into the language of engineering;

(5) Complete the engineering to the stage of blueprints;

(6) Should the finished product undergo extensive testing then there is a further step in the paper analysis; namely, the evaluation of the test.

For a more complete discussion I refer you to an article published in the Journal of Engineering Education, Nov. 1947, pp. 175–180, (vol. 37, No. 3) entitled "The Mathematical Engineer or Industrial Mathematician" (By K. L. Nielsen).

Mathematics is now truly a profession and high school students should be so informed.

4. Guidance in Steering Youths into Engineering and Mathematics

We come now to my third point: "Guidance in steering youths into engineering and mathematics." When I went to high school most colleges required two years of high school mathematics for entrance into a college course no matter what the curriculum; and as a result most high schools had the same requirement for a

high school diploma. Shortly after that it was felt in many circles that this was too stiff a requirement for a diploma and many high schools dropped any requirement for mathematics. Some colleges followed suit, and it was possible to enroll in a university without having any high school mathematics. Then the war came along and with it the great emphasis on mathematics. The high schools now changed back and leaned too far in the other direction, and some required as much as four years of mathematics for a diploma. I think it is about time we stop kicking mathematics requirements around like a football.

I believe that the solution to the problem is a guidance program which is now becoming more and more possible with the consolidation of high schools so that the real small high school is no longer in existence. I do not believe that it is necessary for every high school graduate to have completed courses in mathematics; but I do believe *that those high school students who are going on to college and may enter engineering schools should be guided into as much mathematics as a normal curriculum will stand.* The last time I was teaching at a university, which was not so long ago, I taught a course in freshman algebra for engineers, and in that class I had students who had not had any high school mathematics. We must prevent such situations by providing proper guidance so that students can come to the university properly prepared for their chosen field of study.

Such a guidance program also affords an excellent place to point out the usefulness of, the necessity for, and opportunities in the field of mathematics.

5. The Teaching of Fundamentals

My fourth and last topic is the teaching of fundamentals in high school mathematics. Before entering into a discussion of this topic I want to make it very clear that I am not opposed to any of the so-called progressive methods of teaching. I am as progressive and futuristic as any individual in this room. I am doing research on instruments for supersonic flight and guided missiles. Some of the ideas that I am working on would put Buck Rogers to shame. I sincerely believe that the teaching of mathematics should go forward, new and better ideas and methods should be incorporated into all teaching.

However, from out of the wilderness, my small voice raises this pertinent question:

Aren't we getting too far away from the real teaching of fundamentals in our secondary mathematics courses?

The ability to *retain* a working knowledge of the fundamental operations and concepts seems to be fading. Is that because we seem to be spending more and more time on the beautification of mathematics and thus we merely expose the students to the subject matter rather than teach it to them?

In order to *retain* a working knowledge of the operations the student must do and do and do those operations; in other words, drill and drill. It certainly isn't pleasant. However, when I was teaching in the university I often wondered why it was that so many students could no longer perform the simple mathematical operations that they, obviously, could do at one time while they were studying the course in high school. When I was teaching analytical geometry I found that the students had forgotten algebra and trigonometry; when I taught the Calculus, I found that students had forgotten analytical geometry, trigonometry and algebra. By the time they got to differential equations they had forgotten everything. During the last few years in which I have been associated with engineers and research scientists I am shocked to see the very same thing true about them. They are unable to perform correctly many of the simple mathematical operations that they learned in college.

Is it because we don't drill enough? Where is our teaching failing in that the students cannot *retain* their knowledge,

even those mathematical steps which should be automatic?

I shall close with an interesting example. I had working for me a statistician who had a college degree in mathematics and all the qualifications to perform the ordinary duties of a statistician. He did a considerable amount of analysis of data, computing standard deviations, correlation coefficients, etc. One day he arrived at an obviously erroneous answer but a check and recheck couldn't locate the error. When I reviewed his work I discovered that he had taken the $\sqrt{(a+b)}$ to be $\sqrt{a}+\sqrt{b}$. After having the error pointed out to him he was still of the opinion that such an operation was correct—he had been doing it for years.

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Demonstration of Acceptance Sampling Plans*

By Mason E. Wescott
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Introduction

Most people think of "sampling" as a technique whereby some useful inference can be made concerning a *whole* by considering evidence observed in a *portion*, or *sample*, from the whole. From the simple act of cutting a plug from a watermelon all the way up to the more complicated procedures of a Hooper Rating, Hit Parade, or Gallup Public Opinion Poll we see and are familiar with this attempt to estimate the nature of things unseen on the basis of a peek at, or sample of, these things.

An industrial enterprise purchasing goods, raw material, or partially processed product from outside suppliers expects the items it buys to conform to certain standards specified in the purchase contract. How may it obtain assurance that these standards are being met, that the money is buying the desired characteristics agreed on in the transaction? Must every incoming piece be inspected?

Of course, one can readily find out how many matches in a box of matches are good matches by lighting (or trying to light) each match! Industry calls this sort of thing "100% inspection." Besides the obvious weakness of leaving no usable items when the inspection test is destructive, as in the case of the matches, 100% inspection is both prohibitively expensive and time consuming. Worst of all, it is very frequently untrustworthy, because of sheer fatigue on the part of the inspector who is faced with the monotony of looking at every bolt, for example, in a shipment of say, 600,000 bolts.

Certainly it would be cheaper, and maybe actually *better* to pass judgment on the incoming material by inspecting *part* of the material.

Maybe, but not necessarily! If the purchaser cannot afford to be wrong, he cannot use sampling, for sampling *always* involves the risk of making a wrong decision. Mr. Gallup recently found this out the hard way, but I imagine all of us at one time or another have had personal experiences testifying to the fact that basing decisions on the evidence of samples can easily lead to making a *wrong* decision.

Nevertheless, from the point of view of industrial practice, purchasing and using as it does, millions of items a month to keep the wheels of commerce rolling, the possibility of passing judgment on the acceptability of incoming material by sampling it rather than 100-percenting it appears attractive despite the recognized inevitability of the risk involved.

As far as industry is concerned, the question is not "Shall we or shall we not do acceptance sampling?" The question is, "*How* shall we sample for acceptance?"

Modern, *scientific*, acceptance sampling plans are at present the best answer to that question. These plans are based on sound mathematical and probability theory. Wide experience in their use has amply demonstrated their ability to give the predicted protection at inspection costs that are economical and consistent with the risks that can be tolerated.

Fundamental Assumptions

Before we undertake to demonstrate some of these plans, or to discuss any of the mathematics behind them, let us note five fundamental postulates that stand *behind the mathematics*.

1. The plans apply only to samples taken from a finite aggregate, or "lot," of discrete items. They do not apply to bulk material, such as a drum of oil, a car of

* Paper read at the Christmas meeting of the National Council of Teachers of Mathematics at Columbus, Ohio. December 29, 1948.

iron ore, or a sack of wheat.

2. The inspection test is such that items subjected to the test come out classified as either good or bad, acceptable or rejectable. Items are not to be identified by these plans at variable points along a continuous scale of values; they are to be identified as either possessing or not possessing some specified attribute. In other words, it is not the actual diameter of a shaft that matters to these plans; the question is only whether or not the shaft will go through a test hole of standard size.

3. The sample to be inspected shall be a random sample. This means either that the items in the lot are thoroughly mixed before the sample is taken, or else that the items chosen for the sample must be selected without bias from all parts of the lot.

4. It is not the purpose of the plan to decide the *quality* of the particular lot from which the sample came. The purpose of the plan is to provide the inspector with objective criteria by which he can decide how to *dispose* of the lot, i.e., accept or reject it. This sounds contradictory, because how can an inspector *accept* a lot unless he knows it to be a *good* lot or *reject* a lot unless he knows it to be a *bad* lot? This is precisely where the risks of sampling come in, because only very rough estimates can be made of lot quality on the basis of sample evidence. Hence, the mathematics behind these plans abandons the futile attempt to assess individual lot quality and purposely makes allowance for the fact that the plans will sometimes direct an inspector to make the *wrong* decision. This leads us to postulate 5.

5. The plans are designed to provide *average* lot quality *protection* over a *series* of submitted lots, not quality *determination* for an *individual* lot. This means that if all you are going to be concerned with is a *single lot*, you will have to inspect every item in the lot if it is important to know what the quality of that single lot is. However, if you are going to have lot after lot coming in day after day from different suppliers, and if you can afford to run some risk of rejecting some good lots and accepting some bad lots, then it is money in your pocket to use a scientific acceptance sampling plan that has built into it the ability to perform according to the general scheme shown in the table below.

Operating Characteristics

These performance features of a scientific lot-by-lot acceptance sampling plan can be pictured graphically by an "operating characteristic," or "OC," curve, For *perfect* 100% inspection the *OC* "curve" can be pictured as in Figure 1. Since *sampling* is not present in this case, no risk of making a wrong decision is theoretically possible, so this type of acceptance inspection classifies all submitted lots as good or bad, accepting all that are good and rejecting all that are bad. The point of division between these two classes is called the "AQL," or "acceptable quality level," and is specified by the consumer in terms of per cent defective, for example 4%.

For a scientific, lot-by-lot acceptance *sampling* plan, the OC curve pictures the expected performance of the plan over a *series* of submitted lots of varying quality as shown in Figure 2. Note how this curve exhibits the way the laws of probability operate over the three different classes of lots submitted for acceptance under the

INCOMING LOT QUALITY	SAMPLING PLAN DESIGNED TO
Very good lots	Accept nearly all such lots
Very poor lots	Accept very few such lots
Lots that are neither very good nor very bad—"marginal" lots	Accept a progressively smaller proportion of such lots as lot quality gets worse

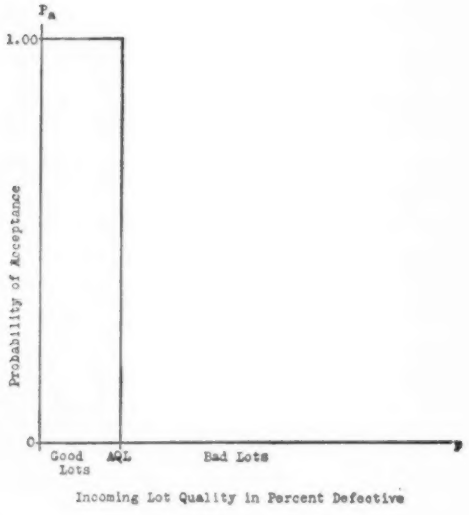

FIG. 1

given plan: very good, marginal, and very bad. The curve shows how the plan can be expected to accept practically all of the very good lots, practically none of the very bad lots, and progressively smaller proportions of lots with marginal quality as lot quality gets worse.

The AQL for the plan pictured in Figure 2 is 4%, and we can expect that 95% of all lots that come in at this quality will be passed by the plan. In other words, the plan subjects the consumer to a 5% risk of making the wrong decision (rejecting) an acceptable (4%) lot.

The division point between marginal and very bad lots is called the "LTPD," or "lot tolerance per cent defective." It is very frequently (as in this plan) associated with a 10% risk. The *OC* curve in Figure 2 shows a plan having a 10% risk at an LTPD of 13.6%. This means that the plan pictured subjects the consumer to a 10% risk of making the wrong decision (accepting) an *un*acceptable (13.6%) lot.

DEMONSTRATION PLANS

We are now ready to see how some of these plans operate. Our lot of "material" will consist of 750 wooden beads 10 mm in diameter. Some of the beads are red—the "defective" pieces in the lot; the remainder are white—the "good" pieces in the lot. To get a "sample" from the lot we scoop up some beads in a wooden paddle full of holes just big enough to accommodate the beads. Since the beads are thoroughly mixed, any sample we scoop up without intentional selection will be a random sample.

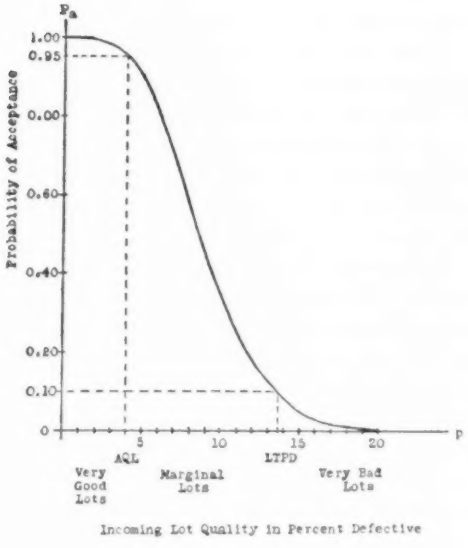

FIG. 2

We shall suppose that our AQL for the material being submitted is 4% defective, and that our LTPD for this material is 13.6% defective so that we shall want sampling plans whose *OC* curves match closely the one we have been discussing.

On page 92 of the Navy Inspection Tables, Appendix X, we find three different sampling plans whose *OC* curves closely approximate our requirements, and which are appropriate for an AQL of 4% and lots of size 750.

Plan 1.—Single Sampling. Here sample size is $n=75$, acceptance number $c=6$. We are to take a sample of 75 pieces, inspect each piece, and count the number of defectives found. If we find 6 or fewer de-

fectives, we accept the lot; if we find 7 or more, we reject the lot.

For the purpose of the demonstrations to be made a data sheet should show headings as follows:

Lot Number	Defectives Found in Sample d	Decision	
		A	R

Plan 2.—Double Sampling. Here the first sample size is $n_1 = 50$, first acceptance number $c_1 = 3$, second sample size $n_2 = 150$, second acceptance number $c_2 = 9$. We are to take a sample of 50 pieces and inspect them. If we find 3 or fewer defectives, we accept the lot; if we find 10 or more defectives, we reject the lot. If we find more than 3 but not more than 9, we take a second sample of 150 pieces and inspect this second sample. If, by the time we have completed the inspection of all 200 pieces (50+150), we have found a *total* of 9 or fewer defectives, we accept the lot; if the total is 10 or more, we reject the lot.

Our data sheet needs the following headings:

Lot Number	Defectives Found in First Sample d_1	Defectives Found in Second Sample d_2	Total Defect. d_1+d_2	Decision	
				A	R

Plan 3.—Group Sequential. Here we take successive samples of 20 each, inspecting each group as we take it, and following the accompanying criteria table to reach a decision on disposition of the lot.

Record in the appropriate column the observed Cumulative Defectives (d) found. As long as this number stays between those in the columns to the left and right of it at any stage, we have to take another sample of 20 and continue inspection. If at any stage d is less than or equal to the Acceptance Number at that stage, we terminate sampling and accept the lot. If at any stage d is equal to or greater than the Rejection Number at that stage, we terminate sampling and reject the lot. A decision is necessarily reached by the end of 7 samples of 20 pieces each—a total of 140 pieces at most.

Our data sheet may be arranged as shown at top of following page.

Demonstration Experiments

To illustrate how these plans may be expected to operate, we shall conduct 9 sampling experiments, three on lots at the AQL of 4%, three on lots at the LTPD of 13.6%, and three on marginal lots each at 8% defective.

Experiment 1. Use Plan 1 on 20 lots each 4% defective. These are acceptable lots, so we do not want our plan to lead us to reject very many of them. Twenty lots represent a fairly short series—"chance" does not have the opportunity to "level off" that would be possible over a series of, say, 200 lots. Nevertheless, the plan

Sample Number	Sample Size	Cumulative Sample	Acceptance Number	Cumulative Defectives d	Rejection Number
1	20	20	0		4
2	20	40	1		5
3	20	60	3		6
4	20	80	5		8
5	20	100	8		10
6	20	120	9		11
7	20	140	10		11

Cumulative Sample	Acceptance Number	Cumulative d in Samples from Lot No.						Rejection Number
		1	2	3	...	19	20	
20	0							4
40	1							5
60	3							6
80	5							8
100	8							10
120	9							11
140	10							11
Decision (A or R)								

should accept at least 16 of these lots, and is more likely to pass 19 or 20 of them. Note that we will have inspected $(20)(75) = 1500$ pieces in passing judgment on what to do with these 20 lots.

Experiment 2. Use Plan 2 on 20 lots each 4% defective. As before (since all the plans used in these experiments have essentially the same *OC* curve), we may expect the plan to pass at least 16 of these lots, and probably 19 or all 20 of them. Were any second samples required to reach a decision? At the end of the experiment, add up the total number of pieces inspected, and divide by 20 to get the "average sample number," called in the literature "ASN." How does it compare with the 75 required for each lot by Plan 1?

Experiment 3. Use Plan 3 on 20 lots each 4% defective. Once more, 16–20 of these lots should be passed by the plan. How many lots were accepted on the evidence of just the first 20 pieces? the first 40 pieces? Find the total number of pieces inspected and divide by 20 to get the ASN. How does it compare with the 75 of Plan 1 and the ASN found for the 20 lots under Plan 2?

Experiment 4. Use Plan 1 on 20 lots each 13.6% defective. These are unacceptable lots, so we do not want our plan to lead us to accept very many of them. Again, 20 lots is not a very long run, but we may expect the plan to pass no more than 6 of these lots, and probably not more than 2 of them. Note that the ASN is again 75.

Experiment 5. Use Plan 2 on 20 lots each 13.6% defective. The plan should not pass more than 5 of these lots, and probably not more than 2. Notice how often a second sample is required. Find the ASN and compare with Experiment 4 and Experiment 2. What seems to be the moral here?

Experiment 6. Use Plan 3 on 20 lots each 13.6% defective. Not more than 5 lots should get by, and probably closer to 2 lots. Note how often the decision to reject is reached quickly. Find the ASN and compare with preceding experiments. What conclusions seem to be justified?

Experiment 7. Use Plan 1 on 20 lots each 8% defective. These are marginal lots, so we do not want our plan to accept too many nor reject too many. In the long run the plan should accept an average of 10–12 lots out of every 20 lots submitted at 8%. But a *single* set of 20 lots is *not* a very "long run," and in any given set of just 20 such lots acceptances can be expected to vary by chance alone all the way from 4 to 18 lots. No matter how many lots are passed, it still requires an ASN of 75 to get a decision under this plan.

Experiment 8. Use Plan 2 on 20 lots each 8% defective. Because these lots are *marginal* in quality, the plan has a harder time making up its mind whether to accept or reject them. Anywhere from 2–16 lots may get by, with 9 or 10 as the more likely number of acceptances. On these 20 lots we shall be taking second samples very frequently. Anywhere from 4 to 18 of these lots can be expected to require a second sample before a decision can be reached; 10 to 12 second samples is the most probable expectation. Find the ASN

and compare with Experiments 7, 5, 4, 2, and 1. What can you say about the expected inspection cost of double sampling for lots of marginal quality as compared with single-sampling such lots and as compared with either single or double sampling for very good and for very bad lots?

Experiment 9. Use Plan 3 on 20 lots each at 8% defective. We can expect to accept anywhere from 4 to 18 of these lots, with 9–12 the more likely number. Note the ASN again and compare with preceding experiments. What conclusions appear to be justified concerning the relative inspection cost of group sequential?

Summary

These nine experiments should supply evidence to demonstrate

1. the relative *equality of protection* furnished by three different *types* of sampling plans which have been designed to have approximately identical *OC* curves;

2. the folly of using double sampling for lots from a supplier who submits material of marginal quality: too many second samples are required, and hence inspection costs rise;

3. the relative superiority in point of ASN of group sequential over either single or double plans at all levels of incoming quality when the *OC* curves of all three types are comparable.

One final comment: these experiments have all been made with lots whose exact quality we knew before any samples were taken. At a receiving inspection station in an industrial plant the inspector never knows the quality of the lots inspected for acceptance either before or *after* the sampling. How can he know that he is using the plan on a 4% lot or a 13.6% lot or an 8% lot? The answer is that the inspector *doesn't* know and doesn't *need* to know! All he has to know is what the *OC* curve for the plan looks like. This is the crystal ball that reveals what protection can be expected from a sampling plan when it is applied to a *series* of lots of unknown quality submitted for acceptance.

Scientific acceptance plans give predictable and dependable protection when used as intended by the mathematics built into their design. That is why they are to be preferred to rule-of-thumb procedures that either attempt to do the impossible or at best give only the form but not the substance of real quality protection.

CONCERNING SUBSCRIPTIONS THAT EXPIRE IN MAY

Some of our subscribers have complained because we have sent them notices about renewing their subscriptions which expire in May 1949. They say that their membership in the Council runs until October. This is true. But there are no issues of THE MATHEMATICS TEACHER between May and October. Therefore, these renewal notices are sent in order to avoid some subscribers missing the October issue because of their late renewals.—EDITOR

For Non-Geniuses Only

By H. C. TRIMBLE
Tallahassee, Fla.

THE BELIEF, either expressed or implied, that their most important job is the location of budding scientists is widespread among teachers of mathematics. That this task is a socially significant one, I shall not question let alone deny. The thesis explored here concerns another group of students. The "second track" group, as we have learned to call them, can be led to find in mathematics a language which extends their analytical powers.

General mathematics has been interpreted too often as an opportunity to teach as many topics from the field of mathematics as can be reasonably concentrated into the allotted time. The text may be organized around social applications, order of difficulty of topics, problems in an historical setting, or the like. There is a danger that a student who masters the course may emerge with a residue of isolated skills and understandings, applicable only to the kinds of exercises which happened to arise in his or her particular course. Experience shows that such specific skills and understandings deteriorate rapidly unless regularly used. Often a lapse of a few years seems to produce complete forgetting.

The need for a central idea about which to organize courses in mathematics for general education seems apparent. The most inspiring writing which I have seen in this connection appears in *Mathematics in General Education*.[1] This "Report of the Committee on the Function of Mathematics in General Education" expresses my own philosophical point of view but stops short of giving me one simple idea about which to begin to organize a "second track" course.

The idea which has caught and holds my attention is "To most people, mathematics is primarily a language for expressing certain sorts of ideas." There is motivation in this idea. A quiz kid may unravel a perplexing problem "just by the sense of the thing." One lacking his excellent insight needs a language in which he can express the problem in a more comprehensible way and proceed step by step to a solution. Given such a language, a non-genius may pass up the quiz kid as the tortoise passed the hare in the well known fable.

Rather than expound this theme at length in general verbal terms, let me take an example. Such examples are plentiful. Our mathematics texts are full of them. Mine is taken from the field of geometry and written for teachers of high school mathematics. The point to be made is, simply, "To the extent that you fall short of genius, you must make up this lack by a command of language tools."

One evening a few weeks ago I was reading a curriculum bulletin published in Florida for secondary mathematics teachers.[2] A picture of a pantograph caught my eye, and I began to wonder about the principle of its operation. It was late and I was tired. The text *Field Work in Mathematics*[3] gave me the fact I wanted but no proof. On page 137 one finds "If in any parallelogram $ABCD$ the four sides, produced if necessary, are cut by a straight line, the four points thus determined will continue to be in a straight line no matter how much the angles of the parallelogram are changed. Furthermore, the distances between these points will remain in a constant ratio to each other."

[1] Commission on Secondary School Curriculum, *Mathematics in General Education* (New York, D. Appleton-Century Company, 1940).

[2] State Department of Education, Tallahassee, Florida, Bulletin No. 50, "A Brief Guide to Teaching Mathematics in the Secondary Schools," 1946.

[3] Shuster, C. N. and Bedford, F. L., *Field Work in Mathematics* (New York, American Book Company, 1935).

That evening my synthetic geometry failed me completely. Only a recourse to analytical methods, to a more powerful approach through language, gave me the release from tension that set me up for a sound night's sleep.

The analytical proof is easy if you have the language ready to be used. Such proofs do not require genius. Ordinary people, even late at night, can make such proofs if they speak an appropriate language. One puts the problem into the machine. One turns the algebraic crank. Out comes the solution.

The language which helped me is in every elementary analytic geometry text. It is often buried in an isolated section concerned with dividing a line segment in a definite ratio. Some treatments of analytic geometry[4] make extensive use of the idea to express equations of straight lines in parametric form. For the present purposes the result may be expressed as follows:

Lemma

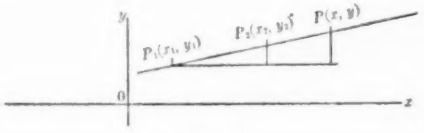

FIG. 1

$$\text{If } \frac{P_1P}{P_1P_2}=k, \quad \text{then} \quad \begin{aligned} x&=x_1+k(x_2-x_1) \\ y&=y_1+k(y_2-y_1). \end{aligned}$$

Provided only $P_1 \neq P_2$, the coordinates of every point, P, on P_1P_2 are expressible in this form, and conversely, to each value of k is determined a unique point P on P_1P_2. The proof by similar triangles is obvious. It should be noted that $k<0$ gives points to the left of P_1, $k=0$ gives P_1, $0<k<1$ gives points between P_1 and P_2, $k=1$ gives P_2, and $k>1$ gives points to the right of P_2 in the diagram.

Any parallelogram may be placed on a set of coordinate axes as indicated in the accompanying figure. There is no loss of generality in placing one vertex at the origin, 0, and a second at $0'$ $(1, 0)$, by using the base of the parallelogram as the unit of measure. In deforming the angles of the parallelogram, these two vertices may be held fixed. An arbitrary fixed point $Q_0(a, 0)$ is chosen on $00'$.

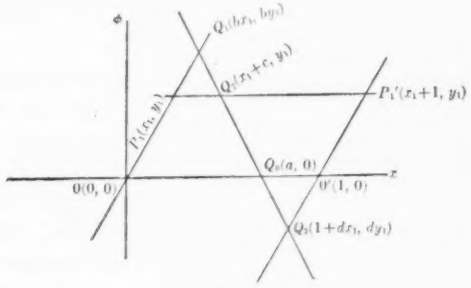

FIG. 2

For a particular position of $P_1(x_1, y_1)$, the coordinates of P_1', Q_1, Q_2, and Q_3 are written down by repeated application of the Lemma. Thus, assuming

$$\frac{OQ_1}{OP_1}=b, Q_1 \text{ has coordinates } \begin{aligned} x&=0+b(x_1-0) \\ y&=0+b(y_1-0). \end{aligned}$$

The numbers a, b, c, and d become definite, fixed numbers once the straight line Q_0Q_1 is drawn. Each expresses a ratio of two segments of its side of the parallelogram. The question of proving the theorem becomes one of showing that Q_0Q_1 continues to pass through Q_2 and Q_3 as P_1 is moved, and that the ratios of segments of the line Q_0Q_1 are invariant.

The coordinates of any point on Q_0Q_1 are expressible as

$$\begin{aligned} x&=a+t(bx_1-a) \\ y&=\quad t(by_1). \end{aligned}$$

To get $Q_2(x_1+c, y_1)$, the second of these equations requires that $t=t_2$ where $y_1=t_2by_1$. Provided $y_1 \neq 0$, and $b \neq 0$, $t_2=1/b$. Since t_2 is independent of x_1 and y_1, the point Q_2 remains collinear with Q_0 and Q_1 as P_1 moves. Moreover, the ratio

$$t_2=\frac{Q_0Q_2}{Q_0Q_1}=\frac{1}{b}$$

[4] Murnaghan, Francis D., *Analytic Geometry*, Prentice-Hall Inc., New York, 1946. Eisenhart, L., *Coordinate Geometry*, Ginn and Company, Boston, 1939.

is fixed. To get $Q_3(1+dx_1, dy_1)$, one must make $t=t_3$ where $dy_1=t_3(by_1)$. Provided $y_1\neq 0$, and $b\neq 0$, $t_3=d/b$. Since t_3 is independent of x_1 and y_1, the point Q_3 remains collinear with Q_0 and Q_1 as P_1 moves. Moreover, the ratio

$$t_3=\frac{Q_0Q_3}{Q_0Q_1}=\frac{d}{b}$$

is fixed.

The restriction $y_1\neq 0$ may be interpreted as the insistence that the parallelogram have its area not zero. The restriction $b\neq 0$ means that $Q_1(bx_1, by_1)$ must be distinct from 0. Actually this restriction is imposed by the choice of notation. If the line segment OP_1 is thought of as originating at P_1 the restriction would become Q_1 distinct from P_1. This notational problem (related to zero divisions) is a weakness of the language being employed which can easily be overcome by a more elegant choice of notation.

The preceding proof without the need for thought may be reexamined in an effort to get "geometric insights." This may lead one to the insight which would have been at once apparent to a genius. The similarity of triangles $Q_1P_1Q_2$ and Q_1OQ_0, together with the fixed lengths OQ_1, P_1Q_1, and OQ_0 make it obvious that Q_2 lies on Q_0Q_1 as P_1 moves. Moreover

$$\frac{Q_0Q_1}{Q_2Q_1}$$

must remain equal to

$$\frac{OQ_1}{P_1Q_1}$$

as P_1 moves. With this start, it is not difficult to work out an analogy for the point Q_3. Still one is plagued by the difficulties of particular placements of the lines in the figure chosen for the purpose of making the proof.

The reader may be as bored by the preceding example as students sometimes get with the mathematical developments which teachers have them study. For some readers a spark of real interest will have been kindled. Let me appeal to both groups at this point. It does happen sometimes, and especially in a well conducted mathematics classroom, that one's interest is caught by a relationship known to hold or only suspected. The ability to convince oneself and others (to prove) that a relationship does or does not hold is closely related to one's command of the language of mathematics.

At least for non-geniuses, I propose as the basic reason for the study of mathematics the opportunity which it affords for extending one's problem solving powers. If this objective is granted, it has important implications for the organization of a mathematics curriculum. It seems to me to imply that *first* one must want to solve a problem. Then one makes additions to one's mathematical vocabulary and to one's kit of skills with a clear cut reason in mind. The language of mathematics extends one's analytical powers.

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How to Develop Critical Thinking about Inter-Group Relations in the Geometry Classroom

By Paul E. Cantonwine
The Lakeside School, Seattle 55, Wash.

Science has made tremendous progress in the last century. It has been able to eliminate many if not all of our old superstitions about nature. It has replaced man's fear of his unknown, natural environment with a knowledge and confidence in his ability to make a reasonable explanation of the happening of nature. This remarkable progress in science and the understanding of natural happenings has happened largely because man has allowed his intellect and reasoning ability to have precedence over his emotions and fears. He has been able to view the scientific facts objectively thus displacing emotional action due to superstition by intelligent action and empirical experimentation based on sound logical reasoning.

We have noted with alarm the failure of our social, economic, and moral progress to keep step with the scientific advance. Possibly one of the big reasons why the progress in the social, economic, and moral fields has been so slow is because we are still basing our decisions and actions, in these fields, on our old prejudices and are not viewing them objectively with critical judgment. It is much easier to go along with the crowd and continue to base our actions upon our emotions or have someone else do our thinking for us.

One of the objectives of the teaching of geometry in the secondary school is to develop logical reasoning and critical thinking so that the many daily decisions which we as American citizens are required to make will result from careful consideration of all the pertinent facts, thus enabling us to live more fruitful lives. We as geometry teachers recognize that developing the ability to reason from certain definitions and assumptions to definite conclusions concerning geometric relationships does not necessarily develop the same ability to reason concerning social relationships.

The following material has been developed to help bring about a greater transfer of logical reasoning and critical thinking to social, economic, and moral problems which face the world today.

Exercise One

Instructions: Read the opening paragraph very carefully and then answer the questions which follow as accurately as you can. You may refer to the paragraph whenever you find it necessary.

The charter of the United Nations states the following as its purposes and principles: 1. To maintain international peace and security, and to that end: to take effective collective measures for the prevention and removal of threats to the peace, and for the suppression of acts of aggression or other breaches of the peace, and to bring about by peaceful means, and in conformity with the principles of justice and international law, adjustment or settlement of international disputes or situations which might lead to a breach of peace; 2. To develop friendly relations among nations based on respect for the principle of equal rights and self-determination of peoples, and to take other appropriate measures to strengthen universal peace; 3. To achieve international cooperation in solving international problems of economic, social, cultural, or humanitarian character, and in promoting and encouraging respect for human rights and for fundamental freedoms for all without distinction as to race, sex, language, or religion; and 4. To be a center for harmonizing the actions of nations in the attainment of these common ends.

Questions: 1. What are the fundamental assumptions stated above? Make a list of them.
2. What terms need clarification by definition? How would you define them?

Using these assumptions and definitions how would you agree with each of the following conclusions? If you agree, disagree, or are undecided give one of the above assumptions or definitions as a reason for your conclusion and explain.

1. The United States should disarm to give the other nations greater confidence in her peaceful intentions. A U D
2. No nation should be regarded as inferior. A U D
3. High protective tariffs should be established by the industrial nations. A U D
4. Immigration restrictions to this country should be more selective. A U D
5. America should discontinue giving financial aid to other nations. A U D
6. America should withdraw her troops from Germany and Japan and let those people live in peace. A U D
7. We should not concern ourselves with the trouble in Palestine or Greece. A U D
8. We should share our secret of the atomic bomb with Russia. A U D
9. We should exchange students with the other nations. A U D
10. We should strive to eliminate prejudice existing in our nation today between groups of people because of race, religion, or nationality. A U D

Exercise Two

Instructions: Read very carefully the following excerpts from the Declaration of Independence and the Constitution of the United States and then answer the questions which follow as accurately as you can. You may refer to the excerpts whenever you find it necessary.

The Declaration of Independence states in part. "We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty, and the pursuit of Happiness. That to secure these rights, Governments are instituted among men, deriving their just powers from the consent of the governed."

The Constitution: Article I. (Amendment I) Congress shall make no law respecting an establishment of religion, or prohibiting the free exercise thereof; or abridging the freedom of speech, or of the press; or the right of the people peaceably to assemble, and to petition the government for a redress of grievances.

Article XIV.—Sec. I. All persons born or naturalized in the United States, and subject to the jurisdiction thereof, are citizens of the United States and of the State wherein they reside. No State shall make or enforce any law which shall abridge the privileges or immunities of citizens of the United States; nor shall any State deprive any person of life, liberty, or property, without due process of law; nor deny to any person within its jurisdiction the equal protection of the laws.

Article XV.—Sec. I. The right of citizens of the United States to vote shall not be denied or abridged by the United States or any State on account of race, color, or previous condition of servitude.

1. What are the assumptions upon which our government is based as given in these excerpts? Make a list of them.
2. What terms need to be defined? How would you define them?

Using these assumptions and definitions how would you agree with each of the following conclusions? If you agree, disagree, or are undecided give one of the above assumptions or definitions as a reason for your answer and explain.

1. Restrictive covenants on real estate in certain areas because of race, creed, or nationality are justified. A U D
2. Every individual is entitled to a job which uses his maximum skill and potentialities. A U D
3. No poll tax or other restrictions, except residence, should be placed on a citizen's right to vote. A U D

4. "White Trade Only" signs in restaurants and hotels should be made illegal. A U D
5. The various groups should be segregated in our public schools. A U D
6. All denominations should be discontinued and everyone accept a common religion. A U D
7. The Ku Klux Klan and similar organizations render a valuable service to our nation A U D
8. The communists should not be allowed the right of public gathering as they are trying to overthrow our form of government. A U D
9. The Jehovah witnesses should not be allowed to hold their gatherings within the city limits. A U D
10. Violence and destruction of property should never be resorted to by the labor unions. A U D

Exercise Three

The three great religions of our nation, Catholic, Protestant, and Jewish, agree pretty well on the following fundamental assumptions: 1. There is one God the Father of Mankind, 2. There is one Brotherhood of all Men, 3. Man should live a life of service guided by high moral and ethical principles, and 4. The dignity and rights of the individual.

Using these assumptions how would you agree with each of the following conclusions? If you agree, disagree, or are undecided give one of the above assumptions as the reason for your conclusion and explain.

1. All men should worship God in the same manner as there is only one true way of worshipping God. A U D
2. A man should be judged mostly by his education and intelligence. A U D
3. Churches should segregate the colored people from the caucasoids. A U D
4. Churches should allow individuals of every race to join in their membership. A U D
5. More opportunities for fellowship among members of different faiths should be provided. A U D
6. Religions which appeal to the emotions through revival meeting should be discouraged. A U D
7. Missionaries should be discouraged in their efforts to show native peoples a more healthful and prosperous way of living. A U D
8. Laws should be established to prevent inter-marriage between Jews and Catholics. A U D
9. Any individual has the right to believe whatever he please and use any means other than violence to encourage others to believe as he does. A U D
10. We as individuals have a right to criticize, discredit, and discriminate against any individual who accepts religious beliefs which we believe to be contrary to God's will. A U D

Exercise Four

Instructions: In the following incidences you are to choose which of the decisions are most in agreement with the postulated principles stated, and give the reason for your choice.

1. An employer has a policy in his large, general merchandise store that all customers should be given courteous and fair treatment by his clerks. A clerk is discourteous to a Japanese-American shopper and refuses to wait on him. The employer may:
   a. Speak to the clerk about the situation and make it clear to her that a recurrence of the incident will bring her dismissal.
   b. Disregard the incident.

c. Tell the clerk that if she does not care to wait on individuals of Japanese ancestry that she should avoid them without being discourteous to them and let someone else wait on them.
d. Discharge the clerk immediately for her action.

2. The United Meat Cutters and Packers Unions' national policy is not to discriminate against a man because of his race, creed, or color and only recognize insubordination and incompatibility on the job or inefficiency as reasons for dismissal. The union sent one of its members to a job but he was not put to work by the employer because he was a negro. The president of the union may:
a. Send another man to that employer.
b. Talk to the employer and ask him to give the man a trial and if he does not prove compatible with the other employees then the union will send the employer another man. If the employer is not wllling to do this then send him another man.
c. Talk to the employer and ask him to give the man a trial and if he does not prove compatible with the other employees then the union will send another man. If the employer is not willing to give the man a trial then threaten to call a strike and picket the place of business.

Exercise Five

Instructions: In the following generalization exercise you will find some statements which you sometimes hear people make and which you sometimes make yourself. The first word of each statement has been omitted. You are to indicate which of the five responses you are sure would most accurately complete the sentence. The five responses are *All*, *Most*, *Don't Know*, *Few*, and *No*. *All* means 100%; without exception. *Most* means more than 50% and less than 100%, *Don't Know* (how many) means that you do not know whether the statement applies to more or less than 50% of the group or figures in question. *Few* means a relatively small number; definitely less than 50%. *No* means not one; 0%.

1. _____ triangles have two sides equal.
2. _____ Japanese-Americans are loyal American citizens.
3. _____ Jews undesirable neighbors.
4. _____ angles are right angles.
5. _____ right angles are equal.
6. _____ lines are curved lines.
7. _____ line segments extend indefinitely.
8. _____ adjacent angles are complementary.
9. _____ complementary angles are adjacent.
10. _____ negroes are dependable workers
11. _____ high school students have below average mental ability.
12. _____ athletes are poor students.
13. _____ redheads have a fiery temper.
14. _____ labor unions are undemocratic.
15. _____ politicians are dishonest.
16. _____ statistics are inaccurate.
17. _____ snakes are poisonous.
18. _____ capitalists are unfair to their employees.
19. _____ indians are clean and homes sanitary.
20. _____ supplementary angles are equal.

EXERCISE SIX

Instructions: Look up the scientific definition of race and apply it in proving the following hypothesis: The Jews are not a Race of People.

Method: The indirect method of Proof.

Proof:

1. The Jews are either a race of people or they are not a race.	1. No other possibility exists.
2. Assume that they are a race. Then (See definition of race)	2.
3. But this is impossible.	3. Members of the three great races, Negroid, Mongoloid, and Caucasoid all accept the Jewish religion.
4. Therefore Jews are not a race of mankind.	4. No other possibility exists for if the conclusion of a correct line of reasoning is shown to be false, then the hypothesis from which the conclusion follows must be false.

GENERAL MATHEMATICS

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By WILLIAM DAVID REEVE

Teachers College, Columbia University

In these three workbooks for the junior high school the placement of each division of mathematics and the emphasis upon each follow closely the recommendations of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics in the Fifteenth Yearbook—*The Place of Mathematics in Secondary Education.*

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The author has had long experience as a high-school teacher, has trained hundreds of mathematics teachers in his graduate classes, and has written many successful texts.

The Odyssey Press **386 Fourth Avenue, New York, N.Y.**

Please mention the MATHEMATICS TEACHER when answering advertisements

THE ART OF TEACHING

Notes on Impossible Problems in Elementary Mathematics

By O. A. GEORGE
Public Junior College, Mason City, Iowa

I WAS interested in the article "Impossible and Unsolved Problems in Elementary Mathematics," by Roy Dubisch, in the October, 1948, issue of THE MATHEMATICS TEACHER. I quite agree with the author in admitting the instructional value of such problems.

Mr. Dubisch cites the problem: "A plane starts out from an airport traveling at 200 miles per hour. Two hours later, a second plane traveling at 300 miles per hour leaves the same airport traveling in the same direction. At what time after leaving the airport will the second plane have traveled twice as far as the first?" Calling the required time t, we have the equation $300t = 2[200(t+2)]$. Solving this equation, $t = -8$. The negative value of t would indicate, as Mr. Dubisch correctly says, the impossibility of the problem, as stated.

However, it is possible to give the seemingly impossible solution a real meaning. Just suppose we assume that the two planes did not start from the airport, but instead had been traveling at their same respective speeds before reaching the airport. Then the solution $t = -8$ would have the meaning that during the 8 hours before the second plane reached the airport, and during the 6 hours before the first plane reached the airport, the second plane had traveled twice as far as the first plane, up to the time the airport was reached.

In proving trigonometric identities, too, I am always wishing authors of textbooks would include a few non-identities.*

* A few of the more recent books include such examples.—Editor.

It has always seemed to me that students would find it much more interesting to approach a set of exercises having the following directions: "Determine which of the following are identities and which are not identities." This would certainly require more skill on the part of the student, and furthermore such an approach would exhibit that puzzle-aspect which usually inspires more interest.

In the trigonometric field there is also the problem of solving a triangle when two sides and the angle opposite one are given, the well known *Ambiguous Case*. For example, if two sides of a triangle are 200 and 300, and the angle opposite the first side is 97°, no solution is possible, because the side opposite 97° would have to be as large as, or larger than, the side 300, as one can easily see by merely drawing a figure. In fact, the Ambiguous Case often gives rise to several impossible triangles.

In the field of progressions, we often run into seemingly impossible situations. For example, in the arithmetic progression in which $d = 4$, $l = 43$, $s = 252$, we get $n = 12$, $a = -1$ for a meaningful solution. But we also get $n = 10\frac{1}{2}$, $a = 5$, and since n represents the number of terms, we think this must be an impossible solution. But, if we let $n = 10\frac{1}{2}$ mean the term midway between the 10th and 11th, that is midway between 41 and 45, say 43, then the problem checks and has some sense. However, to get the required sum, 252, in this case, would be a bit irregular.

Other examples of impossible problems are easily found in elementary mathematics. Thus, the simple equation

$$4+\frac{x+3}{x-3}-\frac{4x_2}{x^2-9}=\frac{x-3}{x+3}$$

has no solution, since the only value of x resulting from normal solving procedures, namely $x=3$, will not satisfy the equation. This is due to the fact that division by zero is excluded from legitimate algebraic operations.

Certain equations containing radicals, such as $3\sqrt{x}=-4$, have no solution. For squaring gives $9x=16$, $x=16/9$, and in attempting to check this value of x, we get the non-identity $4\neq -4$.

Geometric Models Kit

A MATHEMATICS KIT has just been produced by Science Service, the Institution for the Popularization of Science, Washington, D. C. as Unit No. 102 in their "Things of Science" Service and will be distributed to their regular list of subscribers. It consists of 10 specimens needed to make four models for geometry and an explanatory leaflet which suggests 25 experiments that can be performed with the models.

The following statements appear in the introduction to this leaflet:

"So many high school and college students memorize theorems of geometry without actually visualizing what they mean that this series of models was developed for *Things of Science* members with the cooperation of the National Council of Teachers of Mathematics to illustrate a variety of ideas and theorems from plane geometry. If you have already mastered plane geometry, these will refresh your memory; if geometry is still something you will study in the future, these will help you understand some of the fundamentals. And in any case, they are interesting models to play with."

The original plan for this kit was submitted by Miss Frances M. Burns of Oneida, New York, while serving as a member of the Committee on Mathematics Kits of the National Council of Teachers of Mathematics. Additional members of this Committee are: Professor M. H. Ahrendt, Chairman, Anderson College, Anderson, Indiana; Miss Florence Krieger, High School, Rapid City, South Dakota, and Professor Kenneth P. Kidd, University of Florida, Gainesville, Florida.

A limited number of kits, in addition to the 10,000 for Science Service Subscribers, has been prepared and may be obtained by individuals or for use in the mathematics classroom. Orders for use in fall classes should be placed now. The price of the kits is 50 cents each, 3 for $1.00 or $4.00 a dozen. Payment should accompany orders and should be sent directly to Professor M. H. Ahrendt, Anderson College, Anderson, Indiana.

AIDS TO TEACHING

By

HENRY W. SYER
School of Education
Boston University
Boston, Massachusetts

DONOVAN A. JOHNSON
College of Education
University of Minnesota
Minneapolis, Minnesota

BOOKLETS

B. 22—*Computation with Approximate Data*

Carl N. Schuster; State Teachers College; Trenton, New Jersey.

Booklet; 6″×9″; 28 pages; 25¢ each, 15¢ each in lots of 10 or more.

Description: This booklet is a reprint of Dr. Schuster's contribution to the Twentieth Yearbook of the National Council of Teachers of Mathematics, "The Metric System of Weights and Measures." It includes a short historical description of our number system, followed by a discussion of the method of rounding numbers and of performing the fundamental operations (addition, subtraction, multiplication, division, and square root) with approximate numbers. There are exercises with answers given, as well as a helpful bibliography.

Appraisal: This booklet is of just the right length for secondary school classes which need a clear and short introduction to computation with approximate data which is more detailed than the majority of textbooks at the present time, but less than the chapters or sections in advanced engineering textbooks. This introduction is especially valuable and necessary in plane geometry and trigonometry classes and should certainly be given to pupils who have definite plans for advanced work in engineering schools or science. One of the drawbacks is the small number of problems, but these are of enough variety to suggest others which teachers and pupils can concoct. Better still, after a study of approximate computation, a problem based upon actual measurements within the classroom and outside would be even more appropriate than more textbook problems.

B. 23—*A Rhythmic Approach to Mathematics*

Miss L. E. Christman; 1217 Elmdale Avenue; Chicago, Illinois.

Booklet; $4\frac{3}{4}''\times7\frac{1}{4}''$; 68 pages; 50¢.

Description: In this short booklet by Edith L. Somervell, originally published in London in 1906 and reprinted in this country is found a complete, entertaining, artistic, and mathematical description of how curve-stitching on cardboard can be used in teaching. The book has 33 plates of pictures, 22 of them full-page. By sewing in colored thread in systematic fashion from point to point on geometric curves, one can produce patterns which are both artistic and illustrative of mathematical functions. Technically, many of the curves are envelopes, apparent from the placing of the line segments which comprise the stitches. A few typical mathematical curves are the parabola, the curve of pursuit, the spiral of Archimedes, and various loci produced within a circle.

Appraisal: It is hard to describe in words the fascination brought into mathematics by the curve-stitching. At first glance, it seems to be an activity for kindergartens, but this is not where its usefulness stops. By applying this tech-

nique to the types of curves mentioned above and continuing to expand the mathematical explanations which accompany the production one can utilize such stitching in algebraic graphs, geometric loci problems, and even college problems in envelopes. Do not feel that it is an occupation for girls alone—boys will take to it as well.

B. 24—*Just for Fun*

Z. L. Loflin and Ida Mae Heard; Southwestern Louisiana Institute; Lafayette, La.

Mimeographed booklet; $8\frac{1}{2}'' \times 11''$; 55 pages; $1.00.

Description: This booklet, mimeographed on one side of each page, consists of jokes and very short anecdotes which the authors have collected to enliven the teaching of mathematics. They have classified them under Arithmetic (pp. 1–41), Algebra (pp. 42–44), Geometry (pp. 44–51), Trigonometry (p. 52) and Homework (pp. 53–55). There are many line drawings scattered throughout the text to illustrate the jokes.

Appraisal: The idea of having jokes available which teachers will know and feel free to use in mathematics classes is certainly excellent. The authors themselves suggest in their preface that these will prove most valuable to the teacher who uses them incidentally as they are appropriate in teaching, rather than reading them at odd times with the class. The quality of the stories is uneven, for some are very clever and appropriate, while others are quite far-fetched in their application to mathematics. However, it is well to have too large a collection rather than too small for a sense of humor is such an unpredictable quality that jokes seemingly inconsequential to one may be both funny and appropriate to someone else. It is to be hoped that some future collections can be made of jokes even more mathematical, but until then this will serve as a beginning and suggestion.

FILMS

F. 36—*Let's Count*

Coronet Films; Coronet Building; Chicago, Illinois.

Film; 16 mm; 1 reel; black and white, $45.00, color, $90.00; 1948.

Collaborator: F. Lynwood Wren.

Content: This film provides a number of situations in which the counting of objects is necessary. A boy and girl count such objects as baby chicks, marbles, children, blocks, tickets, and tally marks. The difference between ordinal and cardinal numbers is illustrated. Tally marks and numerical symbols are used to determine the total number of groups of objects. It illustrated how the grouping of objects or tally marks aided the counting of these items.

Appraisal: Here is a film that shows how concrete objects can be used to introduce the primary pupil to arithmetic. It should not supplant the use of concrete materials although the pupil viewing the film is asked to count objects pictured and answer the question "How many?" The commentary is different from most educational films in that it is the voice of a woman. This would seem appropriate for a film for primary pupils. The black and white photography sometimes has backgrounds that lack contrast and simplicity. The treatment of the difference between ordinal and cardinal numbers is not adequate. Perhaps the lack of unity and continuity could be explained by the fact that several different levels of attainment were required in the various phases of the film. This would seem especially important at the primary level.

F. 37—*Decimal Fractions*

Johnson Hunt Productions; 1133 North Highland Avenue; Hollywood, California.

Film; 16 mm; 1 reel; black and white, $45.00. color, $85.00; 1948.

Teacher's Guide available.

Content: This film shows the relationship of fractions to decimals, and provides practice in converting fractions into decimal fractions. The film begins by reviewing the meaning of a fraction and the terms of a fraction. Through animation, a cake, divided into ten pieces, is used to illustrate 3/10 and 7/10 as fractions and decimals. The development of the common fraction $\frac{1}{2}$ into .5 is then worked out step by step. Two place decimals are introduced to take care of writing such fractions as $\frac{1}{4}$ and 7/10. One hundred stamps are divided into lots to illustrate the meaning of two place decimals. The film concludes with a review and practice in reading and writing decimals.

Appraisal: Although this film used abstract forms such as disks and blocks and concrete materials such as a cake and stamps to illustrate the meaning of fractions and decimals, most of the explanations are very similar to those of the classroom. It uses materials effectively to show the meaning of fractions and the changing of fractions to decimals. Appropriately, it does not show the changing of a fraction to a decimal by dividing the numerator by the denominator. The limitations of a ten minute film also preclude the inclusion of decimal fractions as an extension of our decimal number system. The omission of live characters may tend to focus the attention of the pupils upon the mathematical operations, but at the same time removes the human interest element. The photography and commentary are satisfactory, but lack interest and uniqueness. The film is recommended for use in the intermediate grades as an introduction to decimals and to the higher grades for remedial instruction. (Reviewed by Charlotte Farm, University of Minnesota High School.)

F. 38—*Meaning of Long Division*

Encyclopedia Britannica Films Inc.; 20 North Wacker Drive; Chicago, Illinois.

Film; Sound, 35 mm; black and white; 1948.

Content: The setting for this picture is a homeroom where the students are studying various lessons. Two members of this room are devising ways and means of illustrating the meaning of long division. The first student shows the solution of $2256 \div 94$ by subtracting 94's until there is no remainder. This subtraction process is also shown by dividing a stick which is 2256 units long into 24 segments. This division process is shortened by increasing the divisor and quotient proportionately. The second student shows the solution of the $7249 \div 23$, first by repeated subtraction and then by repeated division by 2300, 230, and 23. The meaning of a remainder and the long division process are illustrated.

Appraisal: This film uses animated drawings and photographs to emphasize that the division process is a process of repeated subtraction. This concept cannot be taught completely to immature students by a one reel motion picture. However, this film will be a valuable supplement to the class that has previously considered the process. Its greatest value will probably be in teacher education to show teachers of arithmetic how to relate division to the subtraction process. The large numbers used in the problems solved make the illustrations somewhat cumbersome. The commentary and photography are highly satisfactory.

FILMSTRIPS

FS. 51—*Quadrilaterals*

Society for Visual Education; 100 East Ohio Street; Chicago, Illinois.

Filmstrip; 35 mm; black and white; 56 frames; 1948.

Content: Following a series of questions concerning quadrilaterals, polygons are defined and it is explained that the quadrilateral is a member of that family. Sketches and a large number of actual photographs illustrate the general quadrilateral, the parallelogram, the rhombus, the square, the rectangle and the trape-

zoid. The characteristics of each type are covered and the relationships which exist among the various figures are explained by sketches. Examples of included photographs are a garden walk to illustrate the general quadrilateral, a roof to illustrate the parallelogram, a public square to illustrate the square, a tennis court to illustrate the rectangle, and a watering trough to illustrate the isosceles trapezoid. A group of review questions with sketches and their answers complete the filmstrip.

FS. 52—*Parallel Lines and Transversals*

Society for Visual Education; 100 East Ohio Street; Chicago, Illinois.

Filmstrip; 35 mm.; black and white; 46 frames; 1948.

Content: A series of questions introduces the student to the subject. Parallel lines are defined, sketched and photographed to aid the explanation. Photographs of railroad tracks, smoke stacks, and furrows in plowing are examples of the illustrations of parallel lines. The theorem, "If a line is perpendicular to one of two parallel lines, it is perpendicular to the other," is explained by a sketch and the photograph of a building with frontal pillars. The meaning of transversal is also explained with a sketch and is enlarged upon by a photograph of intersecting streets. Interior and exterior angles are shown and all relationships of equality are given by sketches; those relationships involving supplementary angles are omitted. The film concludes with a series of illustrated questions followed by a list of answers.

FS. 53—*Basic Triangles*

Society for Visual Education; 100 East Ohio Street; Chicago, Illinois.

Filmstrip; 35 mm.; black and white; 47 frames; 1948.

Content: After a triangle is defined and illustrated, actual photographs show that triangles often appear in nature as well as in buildings, bridges, broadcasting towers. Triangles are then classified according to angles and sketches and photographs furnish appropriate examples. The parts of a right triangle are also defined by sketches and photographs. Triangles are next classified according to their sides and again sketches and photographs supply examples. After this a series of review questions covering the facts and terms discussed is given. Each of the questions is accompanied by a photograph or sketch illustrating the type of triangle concerned. Finally the answers to the questions are listed.

FS. 54—*Congruent and Overlapping Triangles*

Society for Visual Education; 100 East Ohio Street; Chicago, Illinois.

Filmstrip; 35 mm.; black and white; 48 frames; 1948.

Content: Congruent triangles are defined and illustrated using both sketches and photographs. Natural and man-made illustrations are given (e.g. reflection in a lake and a drawbridge). Corresponding parts are defined and the three basic conditions for congruent triangles are illustrated by sketches. It is shown how one might measure the inaccessible distance across a pond using congruent triangles. The theorem covering the congruency of two right triangles having a hypotenuse and sides respectively equal is shown by sketches and an example of how the theorem might be used in measuring the distance across a stream is given.

For clarity overlapping triangles are shown as they overlap and as they would appear separated. Common sides, common angles and other corresponding parts are indicated. Overlapping triangles are shown in construction and in a lifting device. Median, vertex, midpoint, and altitude are explained by sketches and photographs. Finally a series of illustrated questions appears followed by their answers.

FS. 55—*Areas*

Society for Visual Education; 100 East Ohio Street; Chicago, Illinois.

Filmstrip; 35 mm.; black and white; 49 frames; 1948.

Content: After indicating that in early times the Egyptians used the idea of area in land surveying, the idea of area is explained with a sketch, and a number of situations requiring a knowledge of area are shown. Examples of these are painting a flat surface, seeding farm land, and determining the capacity of an airplane based on wing surface. Square units are explained and sketched and the relationships existing between a square inch, a square foot, and a square yard are indicated. The formulas for the area of a rectangle, a triangle and a parallelogram are given and explained. It is shown that the area of a parallelogram, a rectangle, and a triangle are related. The altitude of a parallelogram and a triangle are sketched. Finally it is shown that the area of a polygon may be found by dividing it into triangles.

FS. 56—*Introduction to Demonstrative Geometry*

Society for Visual Education; 100 East Ohio Street; Chicago, Illinois.

Filmstrip; 35 mm.; black and white; 1948.

Content: In discussing demonstrative geometry, it is explained that one may use postulates, axioms, and theorems in a formal proof. An example of reasoning is given, and examples of postulates, axioms, and theorems are given. Postulates concerning the extension and intersection of straight lines are among those given. The "fundamental operation" axioms are listed. All of these axioms and postulates are accompanied by sketches. The "hypothesis" and "conclusion" are explained, and the "definition" is illustrated using complementary and supplementary angles as examples. The theorem that "straight intersecting lines" form equal vertical angles is proved step by step as an example of formal proof. Finally a series of illustrated questions are accompanied by their answers.

Appraisal of FS. 51–FS. 56: The photography in these filmstrips is generally good, and in most cases quite appropriate pictures are used. There is little repetition among the various strips—a fact in favor of the series as a whole.

The continuity is fairly good but, as is often the case, practicality forces too great a coverage. As a result most of the strips would serve better as a review than as an introduction to the subject matter. The sketches are not always as clear as one might desire. The sketches explaining the axioms are particularly inadequate and drawings in the strip on Area do not show the altitude clearly since these auxiliary lines are not differentiated from boundary lines. Sometimes too much is included on a single frame and sometimes the choice of an example might be improved. The desirability of using a filmstrip to present a proof is questionable. In the opinion of the writer, the continuity of the formal proof presented in this series seemed to be lost. It would seem that the review at the end of each strip might better be undertaken by the teacher. Certainly the inclusion of the answers to the review questions does not use the latter frames to best advantage. In general the coverage of each topic is quite complete and usually numerous and rather clear examples are given. (Reviewed by Theodore Kellogg, University of Minnesota High School.)

INSTRUMENTS

I. 11—*Boyce Meier Sextant* (87697)

Boyce-Meier Equipment Company; Box 281; Bronxville, New York. $4.95.

Description: The light-weight sextant is made of brass and coated with dull black lacquer. The angular scale is approximately $5\frac{1}{2}$ inches long; it has clear white graduations to indicate angles from 0 to 120 degrees. A six-inch index arm contains a vernier which enables the user to determine angular values correct to the nearest six minutes.

Both the stationary and movable mir-

rors are crudely cut and back-silvered but are effective nevertheless. A thin tack within the sighting tube serves as an artificial horizon when the actual horizon is obscured and a bubble-level facilitates holding the instrument properly when the artificial horizon is used. Color filters are provided for reducing glare when the sun is sighted.

Appraisal: An eight-page booklet of directions is included with each instrument. It gives simple instructions for "shooting" the sun and stars, determining one's latitude, and measuring horizontal angles and distance.

The simplified scale of the sextant increases its teaching value; all working parts are unobscured and can be studied better than would be the case with a more intricate device. The student-model is constructed sturdily enough to withstand the effects of handling by inexperienced students but care must be taken to protect the fragile mirrors and bubble level.

Teachers of trigonometry can make excellent use of these instruments which are well worth the price asked. Field trips may well be developed into a project which ncludes drawing simple maps and charts from data gathered with the aid of the sextant. (Reviewed by Bernard Singer, Hyannis, Massachusetts.)

MODELS

M. 6—*Circle Device*

W. M. Welch Manufacturing Company; 1515 Sedgwick Street; Chicago 10, Illinois.

Model; 12″×16″; $8 each for 1 or 2, $6.50 each for 3–9, $6 each for 10–24, $5.50 each for 25–99, $5 each for 100 or more

Description: This new-device developed by John F. Schacht, who is also responsible for the Dynamic Geometry Set (M. 2), is rather hard to describe in words. The metal back with four small feet at the corners has slots through its surface in which a set of arms can slide over the face of the metal background. These three arms are pivoted together and colored green, orange and yellow. They have scales embossed upon them in both directions from the pivot point, running from −1 to 30, −12 to 30, and −12 to 30 respectively. By sliding these arms about, one can arrive inductively at various metric relationships between chords, tangents, and secants of circles. The circumference and diameter of the circle, which is permanently printed on the background are also marked off with scales to facilitate reading.

Appraisal: This device is beautifully made, as are the others which Welch has made available to mathematics teachers. The workmanship is far beyond what amateur teachers or pupils can expect to turn out in making their own models. There is a slight tendency for the sliding section to bind, but nowhere near as much as any similar home-made instrument. The circle device is attractive and adaptable to a large number of different theorems and originals. Its greatest use, of course, will be in classrooms providing one for each pupil or small group of pupils, so that they can use work sheets and discover the statements which they later will prove deductively.

Such a device as this should not be judged hastily by teachers who have not made the necessary adaptations in their whole approach to plane geometry and the accompanying adjustments in their classroom procedure. Such laboratory devices require more than time to be taken from ordinary recitation; they require an attitude of discovery shared by pupils and teachers together. Perhaps such an approach to plane geometry is not appropriate for all classes; for those classes in which it is, devices similar to this are more than valuable—they are essential.

SOURCES OF MATERIAL FOR LABORATORY WORK

SL. 7—*Global Geography*

Scott, Foresman and Company; New York, N. Y.

Workbook and supplementary materials; former is 9½″×12″, 95 pages; latter are

7 sheets about 12″×20″ and one sheet 20″×21″; single set—$2.40 (list price), carton with ten sets—$20.00 (list price).

Description: The workbook consists of 35 lessons with the ones at the beginning the most mathematical. Numbers 1 through 14 deal with such subjects as the following: making hemispheres, flattening hemispheres by separating at a pole or at the equator, making a globe, the grid on a globe, the circular grid used for a Polar Equidistant Map, and mapping on circular grids. The illustrations are large, clear, very useful and appropriate. Each lesson is written in the form of a laboratory procedure with an introductory section, a list of materials and tools needed, and a lesson procedure which includes questions to be filled in.

The eight supplementary sheets are printed on heavy or thin paper as appropriate for their use. There are five sheets of hemispherical gores which are cut out and combined to make a globe, a sheet of cut-out rulers and protractors, an air-section sheet which fits over a finished globe, and a large sheet on which a world air map is to be constructed. The five globes which are put together represent a simple outline map of the world, a political map of the world, an air-current globe, and a globe showing distances from any fixed point on the earth.

Appraisal: By reading through the lessons in the workbook, any mathematics teacher would be convinced that this is the perfect method of correlating mathematics and geography. Activity is not only suggested but absolutely necessary in order to understand the concepts involved. Rulers are curved about the face of the constructed spheres to measure great circle distances; transparent tissue paper hemispheres are slid over the surface of a more substantial globe to show various relationships on the sphere; coordinates are plotted to transfer maps from spherical surfaces to plane surfaces; and, in general, the whole set of exercises is filled with mathematical ideas. The material is priced rather high, but it is certainly well worth it for all who can afford it. It is definitely a great advantage to have a set for each pupil, but rather than forego this excellent teaching aid, school systems could make lesser numbers available to groups of pupils, individual teachers, or the teaching museum in order to illustrate an excellent brand of activity for all levels of mathematics.

Reprints Still Available

Tree of Knowledge	5c
The Science Venerable	5c
Pre-Induction Courses in Mathematics	10c
Coordinating High School and College Mathematics. W. D. Reeve	15c
The Logic of the Indirect Proof in Geometry Analysis, Criticisms and Recommendations. Nathan Lazar	25c
Handbook for Student Teachers and Their Supervisors	25c
Guidance Pamphlet in Mathematics	25c

(In quantities of 10 or more the pamphlet will cost only 10c each.)

The above reprints will be sent postpaid at the prices named. Send remittance with your order to

THE MATHEMATICS TEACHER
525 W. 120th Street, New York, N.Y.

Institute for Teachers of Mathematics

(High School and College)

DUKE UNIVERSITY, DURHAM, N. C.
August 8–19, 1949

W. W. Rankin—*Director*, Professor of Mathematics, Duke University.

Miss Veryl Schult, *Ass't. Director*, Director of Mathematics, Washington, D. C., City Schools.

PROGRAM

(Open discussion following each lecture)

Monday, August 8, 9:00 A.M.–5:00 P.M. Registration
Monday, August 8, 8:00 P.M., West Duke Building.
Presiding—Professor W. W. Rankin
"Simple Arithmetic and Sophisticated Problems"
Dr. Cyril Hoyler, RCA Research Laboratory, Princeton, N. J.
Organization of Study Groups
Miss Veryl Schult, Washington, D. C.

Tuesday, August 9, 11:00 A.M.
Presiding—Miss Mary A. Potter, Racine, Wisconsin.
"Famous Problems and Ideas"
Col. R. C. Yates, Professor of Mathematics, U. S. Military Academy, West Point, N. Y.
Tuesday, August 9, 6:30 P.M.
Banquet—Union Building, West Campus.
Toastmaster—Professor A. M. Proctor, Director, Summer Session, Duke University
Address of Welcome by Dr. A. Hollis Edens, President of Duke University.
Address by Dr. Dwayne Orton, Educational Director, International Business Machine Corporation, New York, N. Y.
Social Hour

Wednesday, August 10, 11:00 A.M.
Presiding—Professor Ruth Stokes, Mathematics, Syracuse University
"The Spirit of Mathematics"
Professor W. H. H. Cowles, Head of Mathematics Department, Pratt Institute, Brooklyn, N. Y.
"Non-Euclidean Geometries"
Professor J. H. Roberts, Director of Graduate Mathematics, Duke University
Wednesday, August 10, 8:00 P.M.
Presiding—Professor C. G. Munford, Mathematics, North Carolina State College, Raleigh, North Carolina.
"Mathematics in Photogrammetry"
Mr. G. C. Tweinkel, Cartographer, U. S. Coast and Geodetic Survey, Washington, D. C.

Thursday, August 11, 11:00 A.M.
Presiding—Miss Sylvia Vopni, Seattle, Washington.
"The Use of Historical Data in Mathematics Classes"
Professor W. W. Rankin, Duke University
Thursday, August 11, 8:00 P.M.
Presiding—Professor W. W. Elliott, Mathematics, Duke University
"Mathematics and Electronic Measuring"
Dr. Ellsworth D. Cook, General Engineering and Consulting Laboratory, General Electric Company, Schenectady, N. Y.

Friday, August 12, 11:00 A.M.
Presiding—Professor J. W. Cell, Mathematics, N. C. State College
"The Development of the New Cadillac High Compression Engine"
Mr. Harry Barr, Cadillac Motor Car Co., Detroit, Michigan.
Friday, August 12, 8:00 P.M.
Reception—University House
Presiding—Dr. C. E. Jordan, Vice President, Duke University
Address: "Applied Mechanics and Progress in Aeronautical Engineering"
Admiral T. C. Lonnquest, Bureau of Aeronautics, U. S. Navy.

Saturday, August 13, 11:00 A.M.
Presiding—Dean W. H. Hall, Engineering College, Duke University
"Mathematics in Automotive Engineering"
Professor L. R. Baker, Director, Chrysler Institute of Engineering, Detroit, Michigan.
Saturday, August 13, 6:30 P.M.
Dinner Meeting—Union, West Campus
Toastmaster—Col. R. C. Yates, Mathematics, U. S. Military Academy
"Coordination of Facts and Fancies"
Mr. E. S. Lee, President American Institute of Electrical Engineers, General Electric Company, Schenectady, N. Y.
Social Hour

Sunday, August 14, 11:00 A.M.
Duke Chapel Service
12:10 P.M., Organ Recital, Miss Mildred Hendrix, Organist, Duke University
3:30 P.M. Carillon Recital, Mr. Anton Brees, Carilloneur, Duke University
Tea—(Rankin Home, 1011 Gloria Avenue), 4:30 P.M.

Monday, August 15, 11:00 A.M.
Presiding—Miss Annie John Williams, Mathematics, Durham High School
"Mechanical Aids in Studying Mathematics"
General W. S. Rumbough, Falls Church High School, Falls Church, Va.
Monday, August 15, 8:00 P.M.
Presiding—Miss Eleanor Rankin, Mathematics, University of Maryland
"Some Applications of Numbers Theory to Elementary Mathematics"
Professor Alfred T. Brauer, Mathematics, University of North Carolina.

Tuesday, August 16, 11:00 A.M.
Presiding—Professor W. H. H. Cowles, Mathematics, Pratt Institute
"Automotive Torsional Vibrations"
Mr. L. F. Hopé, Assistant Head, Mechanical Engineering, General Motors Research Laboratories, Detroit, Michigan
Tuesday, August 16, 4:30 P.M.
Tea—Graham Memorial Building, University of North Carolina, Chapel Hill, N. C.

Dinner Meeting—6:30 P.M., Lenoir Hall, University of N. C.
Toastmaster—Professor J. W. Lasley
"Productive Use of Mathematical Teaching Man Power"
Professor M. W. Whyburn, Chairman Department of Mathematics, University of North Carolina

8:30 P.M. PLANETARIUM BUILDING

Presiding—Professor E. W. Cameron, Mathematics, University of N. C.
"Geometry Overhead"
Professor Toy T. Marshall, Director of Planetarium, University of North Carolina

Wednesday, August 17, 11:00 A.M.
Presiding—Miss Ida Mae Bernhardt, San Marco, Texas.
"From the Student's Seat"
Miss Mary A. Potter, Supervisor of Mathematics, Racine, Wis.
"Aiding the Student in Acquiring Basic Concepts"
Professor W. A. Gager, Mathematics, University of Florida
Wednesday, August 17, 8:00 P.M.
Presiding—Miss Bonnie Cone, Mathematics, Charlotte, N. C.
"A Problem in Quality Control"
Professor A. O. Hickson, Mathematics, Duke University

Thursday, August 18, 11:00 A.M.
Presiding—Miss Christina Little, Boston, Mass.
"Some Problems Associated with Municipal Water Supply"
Professor Brewster Snow, Engineering College, Duke University

Thursday, August 18, 8:00 P.M.
Presiding—Mrs. Nanette Blackiston, Baltimore, Maryland
"Problems and Solutions"
Water Melon Party, 9:30 P.M., Rankin Home

Friday, August 19, 9:00 A.M.
Presiding—Miss Frances Burns, Oneida, N. Y.
"The Beautiful Necessity for Observation"
Miss Veryl Schult, Washington, D. C.
Committee Reports
Adjournment—10:30 A.M.

MATHEMATICS INSTITUTE
Duke University—Aug. 8–19, 1949

Study Groups

I. *Aids in the Study of Algebra*—8:00–9:00 A.M. (Junior and Senior High School) Miss Frances Johnson, Oneonta High School, Oneonta, N. Y.

II. *Outdoor Work in Mathematics*—8:00–9:30 A.M. (Junior and Senior High School) General W. S. Rombough, Falls Church High School, Falls Church, Va.

III. *Modern Geometry with Applications*—8:00–9:00 A.M. (Senior High School and College) Col. R. C. Yates, U. S. Military Academy, West Point, N. Y.

IV. *Arithmetic of Home, Business, and Industry*—8:00–9:00 A.M. (Senior High School).

V. *Graphical Analysis with Applications*—9:30–10:30 A.M. (High School and College) Professor J. W. Cell, N. C. State College, Raleigh, N. C.

VI. *Junior High School Mathematics*—9:30–10:30 A.M. Miss Veryl Schult, Supervisor of Mathematics, Washington, D. C.

VII. *The Slow Student in Mathematics*—9:30–10:30 A.M. (Junior-Senior High School) Miss Mary A. Potter, Supervisor of Mathematics, Racine, Wis.

LECTURE—GENERAL MEETING—11:00–12:00 NOON.

VIII. *Applications of Mathematics*—2:30–3:30 P.M. (High School and College) Professor J. W. Cell, N. C. State College, Raleigh, N. C.

IX. *Laboratory Mathematics* (Fee $2.00)—2:30–4:30 P.M. (Junior-Senior High School) Miss Allene Archer, Thomas Jefferson High School, Richmond, Va.; Miss Frances Burns, Oneida H. S., Oneida, N. Y.

X. *Laboratory Mathematics* (Fee $2.00)—2:30–4:30 P.M. (High School–College) Col. R. C. Yates, U. S. Military Academy, West Point, N. Y.; Miss Amelia Richardson, McKeesport, Pa.

LECTURE—GENERAL MEETING—8:00–9:00 P.M.

CONCERNING SUBSCRIPTIONS THAT EXPIRE IN MAY

Some of our subscribers have complained because we have sent them notices about renewing their subscriptions which expire in May 1949. They say that their membership in the Council runs until October. This is true. But there are no issues of THE MATHEMATICS TEACHER between May and October. Therefore, these renewal notices are sent in order to avoid some subscribers missing the October issue because of their late renewals.—EDITOR

NEWS NOTES

A meeting of mathematics teachers under the sponsorship of the California Mathematics Council was held Saturday March 5th at Stanford University.

2:00 P.M. to 5:00 P.M.

Program:

1. *Speaker*—Doctor George Polya, Stanford University, author of the recently published and highly stimulating book "How To Solve It."
2. Report—Appraisal of the *Guidance Report*, Commission on Post-War Plans.
3. *Discussion*—Northern California's place in the new organization of the California Mathematics Council.
4. *Social Tea.*

The Treasury Department in Washington, (U. S. Savings Bond Division) has issued a Spring 1949 "School Savings Journal for Classroom Teachers."

The *Journal* includes articles on the national welfare, the history of the famous Treasury Bonds displayed on the Freedom Train, education and advertising, and what School Savings offers farm boys and girls. There is a colorful lift-out poster featuring a teacher explaining Savings Bonds to her students.

School Savings this past year has enjoyed healthy growth. Predictable economic problems facing us at home and abroad point to the need for expanding the Program in 1949.

Referring to President Truman's appointment of Earl James McGrath as Commissioner of Education, Oscar R. Ewing, Federal Security Administrator, said recently that Dr. McGrath will enter on his new duties immediately following his confirmation by the Senate. Acting Commission of Education Rall I. Grigsby has been carrying on these duties since the resignation of John W. Studebaker last July.

Born in Buffalo, N. Y., Dr. McGrath graduated from the Buffalo Technical High School in 1920. He received his B.A. degree in 1928 and his M.A. in 1930, both from the University of Buffalo. In 1936, he received his Ph.D. degree from the University of Chicago. The degree of Doctor of Humane Letters was conferred on him by Coe College, Iowa, in 1946.

He has been a member of the National Edution Association for many years and since 1946 has served on the Executive Committee of the Department of Higher Education. Other learned societies of which he is a member include: Phi Beta Kappa; Sigma Xi; Association for the Advancement of Science; Phi Delta Kappa; Delta Chi; Delta Sigma Pi; and Delta Phi Alpha.

Dr. McGrath is Editor of the *Journal of General Education*. He is the author of a number of books on education, including: *Toward General Education; Science in General Education;* and *Social Science in General Education*.

Oscar R. Ewing, Federal Security Administrator, recently announced the appointment of W. Earl Armstrong, of Newark, Delaware, as Associate Chief for Teacher Education on the staff of the Division of Higher Education of the Office of Education, Federal Security Agency.

Dr. Armstrong comes to the Office of Education from the University of Delaware, where he has served as Dean of the School of Education since 1945. He succeeds the late Dr. Benjamin W. Frazier, Office of Education Specialist in Teacher Education for many years.

NEW PUBLICATIONS OF THE OFFICE OF EDUCATION FEDERAL SECURITY AGENCY

Fourteen questions commonly asked about the organization of elementary schools are presented and answered in a new pamphlet just issued by the Federal Security Agency and prepared by specialists in the Office of Education. This publication, Pamphlet No. 105, entitled "14 Questions on Elementary School Organization," gives information based on reports from educators in 52 cities. Copies may be obtained from the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., price 10¢ each.

Another Office of Education publication recently issued deals with the problems of educating crippled children in school. Written by Romaine B. Mackie, Specialist for Schools for the Physically Handicapped, Elementary Education Division, this bulletin entitled "Crippled Children in School" is available for 15¢ from the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. It is Bulletin 1948 No. 5.

Those who plan high school programs and are responsible for guidance for high school age youth should be interested in publications entitled "Postgraduate Education in High Schools, 1947–48" and "Intellectual Abilities in the *Adolescent* Period—Their Growth and Development." Homer Kempfer, Specialist for General Adult and Post-High-School Education, Office of Education, in "Postgraduate Education in High Schools, 1947–48," discusses the organization of the postgraduate program and its place in American education. This Pamphlet, No. 106, may be ordered from the Superintendent of Documents for 10¢.

David Segel, Specialist in Tests and Measurements, Office of Education, studied results of the War Department's aptitude tests adapted for secondary schools and presents, in "Intellectual Abilities in the Adolescent Period—Their Growth and Development," an analysis of his research on the subject of intellectual abilities at the high school level. This is Bulletin 1948, No. 6, and is available from the Superintendent of Documents for 15¢.

TWENTY PER CENT OF TOTAL COLLEGE-UNIVERSITY ENROLLMENT AT TWENTY INSTITUTIONS

Twenty universities enroll 20 per cent of all college students in the United States, according to the Office of Education, Federal security Agency.

College enrollment figures for this fall just released by the Office of Education, based upon returns from practically all of the Nation's 1,800 institutions of higher learning, show a total enrollment of 2,410,000 the peak enrollment in higher education history. Of this number 486,267 are studying at 20 institutions, as follows:

Institution	Total Enrollment
1. New York University	47,647
2. University of California	43,469
3. City College of New York	28,567
4. Columbia University	28,000
5. University of Minnesota	27,243
6. University of Illinois	25,920
7. Ohio State University	23,929
8. Northwestern University	23,788
9. University of Indiana	23,131
10. University of Southern California	22,740
11. University of Wisconsin	22,353
12. University of Michigan	21,002
13. Syracuse University	19,698
14. University of Texas	19,676
15. University of Pittsburgh	19,526
16. University of Pennsylvania	18,644
17. Boston University	18,617
18. Wayne University	18,455
19. Temple University	17,212
20. University of Washington	16,650
Total:	486,267

Largest Enrollments by Institution

The 10 institutions with the largest enrollments reported are: New York University, 47,647; University of California, 43,469; City College of New York, 28,567; Columbia University, 28,000; University of Minnesota, 27,243; University of Illinois, 25,920; Ohio State University, 23,929; Northwestern University, 23,788; University of Indiana, 23,131; University of Southern California, 22,740.

Greater Holding Power

The rate of student drop-out is slowing down, according to the Office of Education survey. This is indicated by the greater holding power of college despite fewer freshmen students and a scaled-down veteran enrollment. Last year's freshmen enrollment stood at 593,000. This year only 569,000 freshmen are reported.

Veterans' Enrollment Down

Veterans, who formed about half of the Nation's college student body in 1947, account for only 42 per cent of the total enrollment in 1948. Delaware, Florida, Maryland and New Mexico are the only States showing increases in number of veteran students this year. Greatest drop in veterans' enrollments came at the junior college level.

Few Institutions Enroll Many Veterans

Approximately 50 per cent of the veterans are enrolled in 131 of the country's large universities. Largest veteran student enrollments are found in the following institutions: New York University, 21,930; University of California, 18,948; University of Southern California, 15,751; University of Minnesota, 14,810; Columbia University, 13,400; Ohio State University, 12,146; University of Illinois, 11,880; University of Wisconsin, 10,808; Northwestern, 10,706; and the University of Pittsburgh, 10,125.

Men Outnumber Women

Men still outnumber women almost 3 to 1 in the Nation's colleges and universities, the Office of Education survey reveals, The proportion of women students has changed little this year over last fall. The 3 to 1 ratio holds in the large universities, although it goes down to slightly less than 2 to 1 in liberal arts colleges, and is more nearly 1 to 1 in teachers colleges. The proportion of women freshmen is slightly higher in 1948 than it was in 1947 in all types of institutions.

The February meeting of the Women's Mathematics Club of Chicago and Vicinity was held on Saturday, February 5, 1949 at 11:45 A.M. in Mandel's Tea Room.

Dr. William C. Krathwohl spoke on "The Work of the Institute of Psychological Services of the Illinois Institute of Technology." Dr. Krathwohl is Director of Tests in the Institute of Psychological Serivces.

WOMEN'S MATHEMATICS CLUB OF CHICAGO AND VICINITY

President—Mrs. Mary Werkman, Parker High School
Vice-President—Miss Bernice von Horn, Hyde Park High School
Secretary—Miss Mae Studer, Zion-Benton, Township High School
Treasurer—Miss Marie Brennecke, Washington High School
Program Chairman—Miss Lenore John, Laboratory School, University of Chicago.

JOINT MEETING OF THE MEN'S AND WOMEN'S MATHEMATICS CLUBS

Time—6:30 P.M., Friday, March 18, 1949
Place—Harding's Presidential Grill, 109 West Madison Street
Speaker—Professor Marshall H. Stone
Topic—"The Discovery and the Cultivation of Mathematical Talent"

The third, fourth, and fifth meetings of the Men's Mathematics Club of Chicago and the Metropolitan Area were held at the following times with respective speakers and their topics:

FALL ENROLLMENT OF COLLEGE STUDENTS BY STATES, 1947 AND 1948

	TOTAL		FIRST-TIME STUDENTS		VETERANS	
	1947	1948	1947	1948	1947	1948
Alabama	32,046	32,597	7,329	7,531	16,241	14,796
Arizona	10,608	11,276	3,428	3,062	4,899	4,874
Arkansas	17,765	18,068	5,803	5,260	8,415	7,766
California	178,663	190,650	45,566	46,852	81,368	74,468
Colorado	32,809	34,278	7,831	9,018	18,235	17,413
Connecticut	27,068	30,129	10,774	7,514	14,807	13,521
Delaware	2,728	3,105	817	828	1,305	1,358
Florida	31,215	34,405	8,817	9,154	15,965	16,174
Georgia	39,634	40,094	10,521	10,483	17,291	15,702
Idaho	8,381	8,242	3,097	2,675	4,034	3,421
Illinois	163,725	152,521	35,451	39,226	75,364	61,519
Indiana	62,686	73,290	15,270	17,611	32,063	29,070
Iowa	43,757	44,952	11,812	11,367	21,725	19,056
Kansas	37,201	37,217	10,952	9,354	17,627	15,637
Kentucky	31,760	32,139	8,594	7,135	14,513	13,289
Louisiana	38,151	37,392	8,881	8,633	16,507	14,898
Maine	9,279	9,544	2,765	2,551	4,768	4,222
Maryland	34,937	37,010	9,529	8,814	14,462	14,588
Massachusetts	93,087	94,961	21,058	20,559	46,250	40,228
Michigan	98,977	101,229	22,974	21,088	48,706	43,726
Minnesota	52,633	51,995	12,344	12,753	25,894	22,568
Mississippi	19,216	19,502	6,380	6,154	9,308	8,269
Missouri	62,274	60,480	19,078	15,311	26,918	25,990
Montana	8,900	8,855	2,619	2,296	4,733	4,207
Nebraska	22,702	22,543	6,714	5,804	11,130	9,589
Nevada	1,964	1,769	681	415	1,008	783
New Hampshire	8,777	8,674	2,584	2,393	4,691	3,708
New Jersey	44,071	42,724	14,176	11,131	24,317	20,107
New Mexico	8,605	9,113	2,409	2,370	4,330	4,452
New York	280,874	308,436	58,417	60,504	127,814	122,733
North Carolina	46,112	45,481	13,976	12,358	20,383	17,763
North Dakota	9,227	8,948	3,092	2,841	4,393	3,826
Ohio	142,279	140,253	33,874	30,235	69,559	62,491
Oklahoma	45,175	44,746	11,495	11,115	23,821	21,719
Oregon	25,520	25,400	6,989	7,244	13,202	12,062
Pennsylvania	142,280	148,949	31,645	28,891	72,484	69,536
Rhode Island	12,397	12,799	4,000	3,757	6,967	6,623
South Carolina	26,119	23,667	7,024	4,048	10,266	9,147
South Dakota	8,678	8,565	2,965	2,679	4,524	3,920
Tennessee	37,312	38,824	11,096	11,153	17,316	15,392
Texas	122,516	126,228	32,277	29,843	60,332	53,533
Utah	20,413	21,769	6,079	6,521	9,463	8,826
Vermont	6,869	7,746	2,322	2,625	2,999	2,951
Virginia	36,455	39,472	10,458	10,703	15,392	14,722
Washington	39,214	41,404	10,050	11,174	20,950	19,687
West Virginia	23,474	22,716	6,539	5,456	11,288	9,543
Wisconsin	49,528	48,559	12,633	10,374	24,544	20,699
Wyoming	3,819	3,990	1,060	979	1,994	1,848
Dist. of Col.	36,346	36,748	8,601	7,396	18,173	17,439
Outlying parts		4,853		1,577		1,182
	2,338,226	2,408,307	592,846	568,815	1,122,738	1,021,041

3RD MEETING—FRIDAY, DECEMBER 17, 6:15 P.M.

Topic—"Speak Your Own Mind."
Moderator—Glenn F. Hewitt, Von Steuben High School and Past President of the club.

Here are some questions which were discussed:

(1) Should there be a terminal course in mathematics for some students?
(2) Should two years of general mathematics be offered and what should be the content of the second year course?
(3) Should minimum standards of mathematics achievement be set for all H.S. graduates?
(4) Should every mathematics course contain a unit of arithmetic?

4TH MEETING—FRIDAY, JANUARY 21, 6:15 P.M.

Topic—"Mother Nature is a Mathematician."
Speaker—Mr. Walter H. Carnahan, High School Mathematics Editor, D. C. Heath & Co.

5TH MEETING—FRIDAY, FEBRUARY 18, 6:15 P.M.

Topic—"The Mathematical Theory of Politics."
Speaker—Professor Harold T. Davis, Chairman

Fall Enrollment in Higher Educational Institutions, 1947 and 1948:
Students Attending College for the First Time by Type of Institution

Type of Institution	Number of First-Time Students		Change	
	Fall 1947	Fall 1948	Number	Per cent
All Institutions	593,000	569,000	−24,000	− 3.0
Universities, colleges and professional schools	408,000	396,000	−12,000	− 3.0
Teachers colleges	55,000	51,000	− 4,000	− 7.3
Junior colleges	107,000	103,000	− 4,000	− 3.8
Negro institutions	23,000	19,000	− 4,000	−17.4

Fall Enrollment in Higher Educational Institutions, 1947 and 1948:
Number of Veterans Enrolled by Type of Institution

Type of Institution	Number of Veterans		Change	
	Fall 1947	Fall 1948	Number	Per cent
All Institutions	1,122,000	1,022,000	−100,000	− 8.9
Universities, colleges and professional schools	950,000	889,000	− 61,000	− 6.4
Teachers colleges	63,000	56,000	− 7,000	−11.2
Junior colleges	83,000	54,000	− 29,000	−34.9
Negro institutions	26,000	23,000	− 3,000	−11.6

of the Department of Mathematics, Northwestern University.

OFFICERS 1948–1949

W. G. Hendershot, *President*, Roosevelt High School, Chicago, Illinois.
David Rappaport, *Sec.-Treas.*, Lane Technical High School, Chicago, Illinois.
H. T. Davis, *Honorary President*, Northwestern University, Evanston, Illinois.
E. L. Tierney, *Rec. Sec.*, Tilden High School, Chicago, Illinois.

Program Committee
Glenn Anderberg, *Chairman*, Waukegan Township High School.
William C. Krathwold, Illinois Institute of Technology.
Joseph A. Nyberg, Hyde Park High School.

Special Project Committee
George F. Bradfield, *Chairman*, Lake View High School.
E. L. Paine, Downers Grove High School.
S. F. Bibb, Illinois Institute of Technology.
C. M. Austin, *Historian*, Oak Park, Illinois.

James Stephen Mikesh, until last June chairman of the mathematics department of The Lawrenceville School, died Saturday morning, January 29, at Mercer Hospital, Trenton, New Jersey, after a brief illness. He was the victim of a stroke the preceding Monday which left him partially paralyzed. His brother, Martin A. Mikesh, was with him at his death. His funeral was held February 2 in Spillville, Iowa.

The forty-sixth annual meeting of the Association of Teachers of Mathematics in New England was held at Boston University.

SATURDAY, DECEMBER 4, 1948

Morning Session

10:30 A.M. Business Meeting and Election of Officers.
10:45 A.M. to 12:15 P.M.
Report on Committee on Examination Subjects by College Entrance Examination Board.
Mathematics on the "Second Track," in High Schools:
I For grade IX.—Miss Margaret E. Macdonald, *Durfee High School, Fall River, Mass.*
II For grades XI and XII—D. Garland B. Russell, *Supt. of Schools, Westboro, Mass.*

Afternoon Session

12:30 P.M. Luncheon.
2:00 P.M. General Mathematics in the College;
I Real Numbers for Freshmen.—Prof. Marie Litzinger, *Mt. Holyoke College.*
II Inquiries in Mathematics.—Prof. Bancroft H. Brown, *Dartmouth College.*
Speech read by Prof. W. F. Cheney, *University of Connecticut.*
Discussion led by Prof. Ralph Beatley, Graduate School of Education, Harvard University.
New officers elected are:
President—Professor Elmer B. Mode, Boston University.
Vice-President—Harold B. Garland, High School of Commerce, Boston.
Secretary-Treasurer—Margaret Cochran, Somerville High School, Somerville, Mass.
Members of the Council—Anna Flaherty, Girl's Latin School, Boston, and Professor D. G. Fulton, Tufts College, Somerville, Mass.